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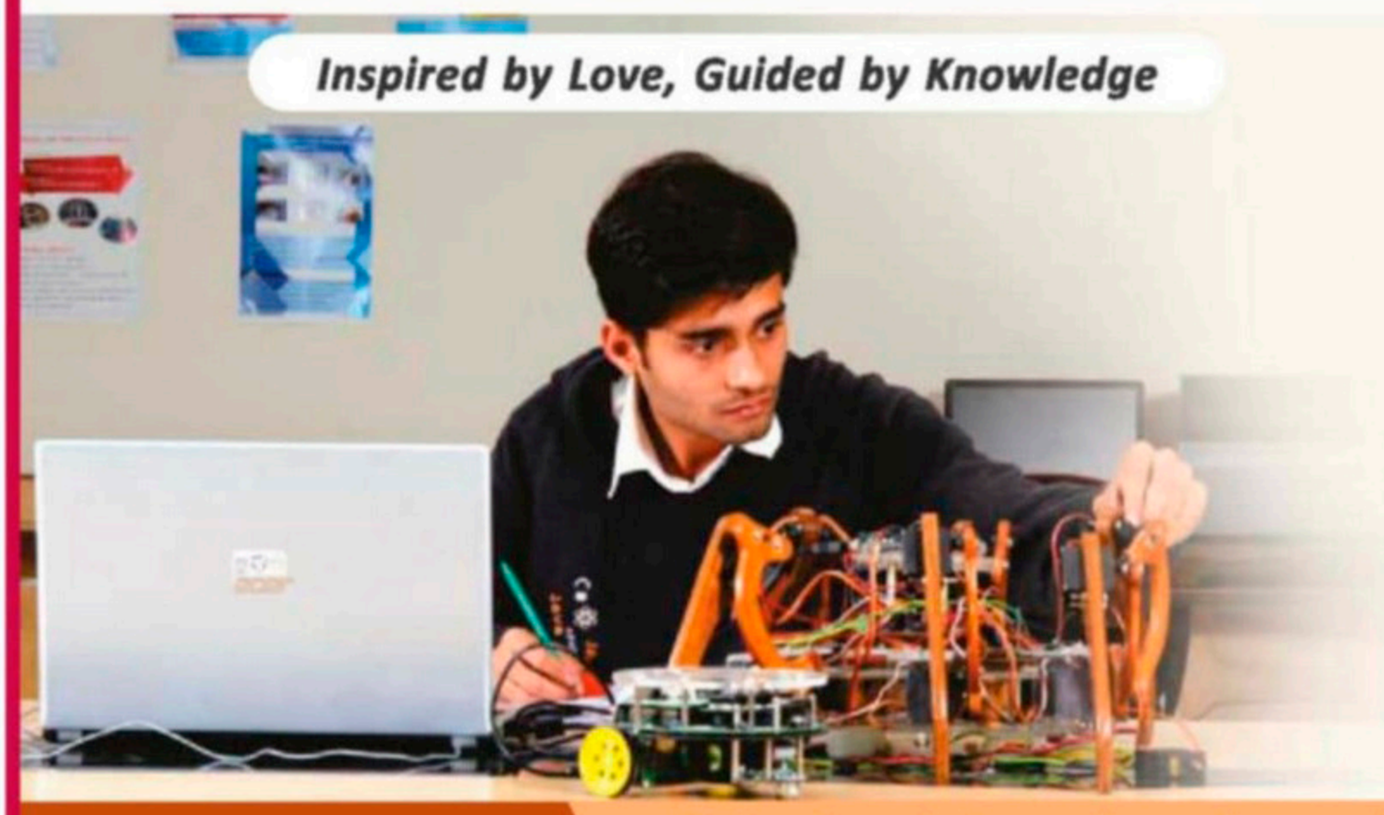

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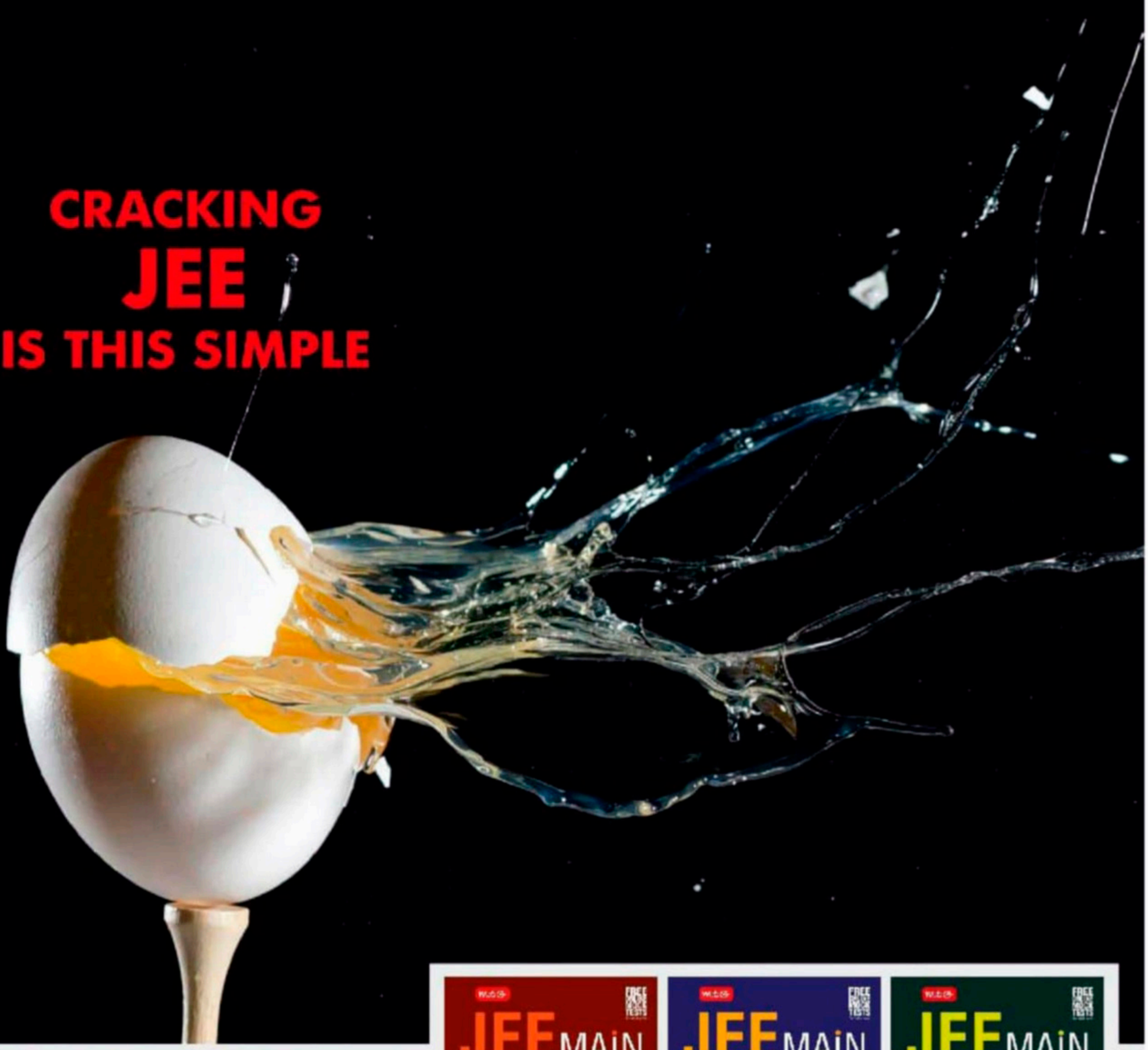

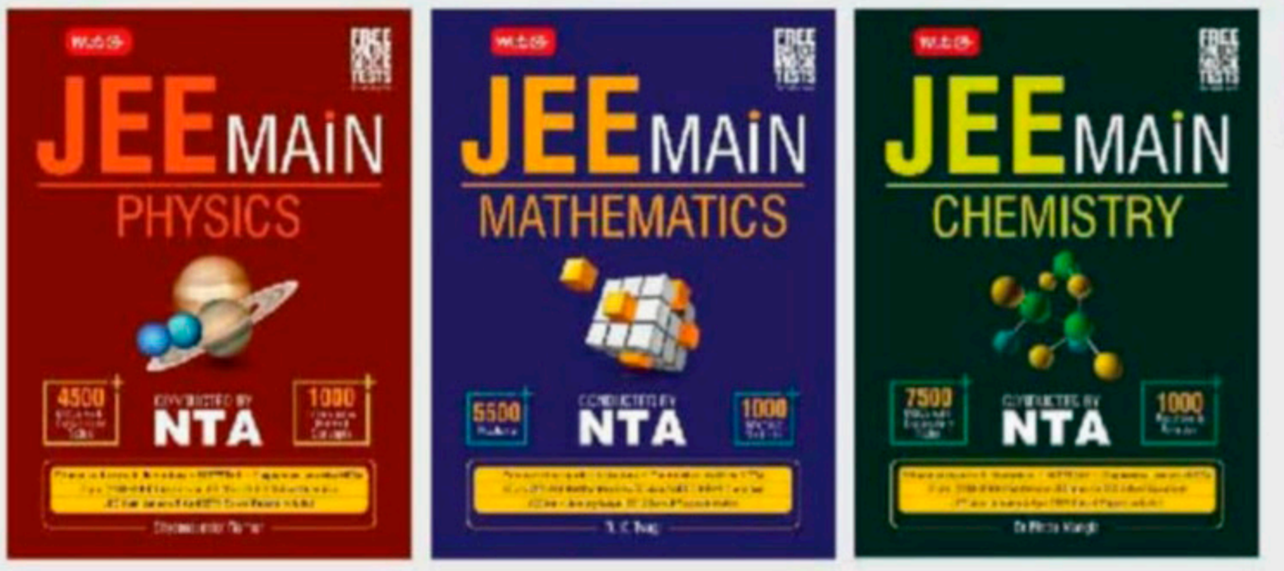

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Vol. XXXVIII No. 1 January 2020

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Managing Editor : Mahabir Singh
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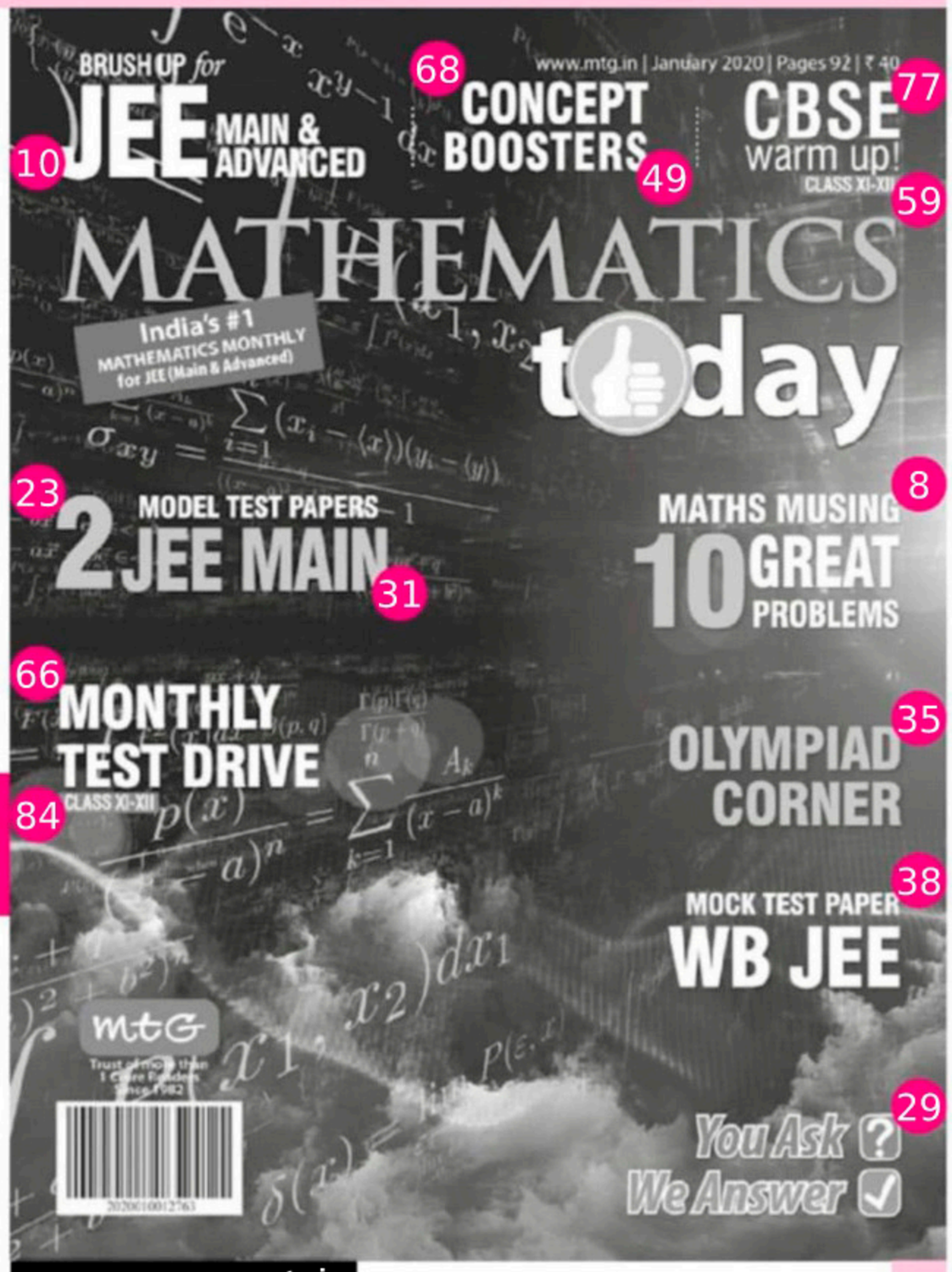

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During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new *pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our* readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 205

JEE MAIN

1. Let $a_1, a_2, a_3, \ldots\ldots$ be a G.P. where $a_1 = a$ and common ratio r are positive integers.
If $\log_8 a_1 + \log_8 a_2 + \ldots\ldots + \log_8 a_{12} = 2014$, then the number of ordered pairs (a, r) is
(a) 44 (b) 45 (c) 46 (d) 47

2. AB is a chord of the circle $x^2 + y^2 = 25$. The tangents at A and B meet at C. If $M(2, 3)$ is the mid-point of AB, then the area (in sq. units) of the kite $OACB$ is
(a) $50\sqrt{\frac{13}{3}}$ (b) $50\sqrt{\frac{3}{13}}$
(c) $50\sqrt{3}$ (d) $\frac{50}{\sqrt{13}}$

3. A five-digit number is formed with the digits 0, 1, 2, 3, 4, 5 without repetition. The probability that it is divisible by 11 is
(a) $\frac{2}{25}$ (b) $\frac{2}{50}$ (c) $\frac{3}{50}$ (d) $\frac{3}{25}$

4. The number of points at which the function $y = f(x) = \left|1 - \left|1 - |x|\right|\right|$ is not differentiable is
(a) 2 (b) 3
(c) 4 (d) 5

5. Let $\{x\} = x - [x]$. The least integer greater than $\int_0^{100} \{\sqrt{x}\}dx$ is
(a) 50 (b) 51 (c) 52 (d) 53

JEE ADVANCED

6. Let $z_1 = 3 - 4i$, $z_2 = 2 + i$, $\alpha \neq 1$, $\alpha^6 = 1$.
The sum $S = \sum_{r=0}^{5} \left|z_1 + z_2\alpha^r\right|^2$ is divisible by
(a) 2 (b) 3 (c) 5 (d) 7

COMPREHENSION

ABC is a triangle right angled at A. Points D and E are on the side AC such that $DC = 20$, $ED = 8$, $\angle ACB = \alpha$, $\angle ADB = 2\alpha$, $\angle AEB = 3\alpha$.

7. $\cos 2\alpha =$
(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ (d) $\frac{5}{6}$

8. $AE =$
(a) 4 (b) 5 (c) 6 (d) 7

NUMERICAL VALUE TYPE

9. Let $|\vec{a}| = 1$, $|\vec{b}| = 4$ and $\vec{a} \times \vec{r} + \vec{b} = \vec{r}$. If projection of $\vec{r}$ along $\vec{a}$ is 2, then the projection of $\vec{r}$ along $\vec{b}$ is $\frac{m}{n}$ in the reduced form, where $m - n$ is

MATRIX MATCH

10. Match the following columns.

	Column-I	Column-II	
P.	If 4 dice are rolled once, the number of ways of getting the sum 10 is	1.	84
Q.	If a, b, c, d are odd positive integers, the number of solutions of the equation $a + b + c + d = 16$ is	2.	55
R.	The coefficient of x^2y in the expansion of $(1 + x + 2y)^5$ is	3.	60
S.	If $C_r = \binom{10}{r}$, then $\sum_{r=1}^{10} \frac{r \cdot C_r}{C_{r-1}}$ is	4.	80

P Q R S
(a) 2 3 1 4
(b) 4 1 3 2
(c) 4 3 2 1
(d) 2 1 3 4

See Solution Set of Maths Musing 204 on page no. 30

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Reality

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Knowledge Quiz - 26

Show that among any 4 numbers one can find 2 numbers so that their difference is divisible by 3.

Please send your detailed solution before 12th January 2020 to quiz@kcseducate.in along with your name, father's name, class, school, address and contact number.

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Unit 7

Matrices and Determinants

MATRICES

- An array of real (or imaginary) numbers arranged in m rows and n columns is called $m \times n$ matrix. $m \times n$ is called the order of the matrix. It is represented by

$$A = [a_{ij}]_{m \times n}, \text{ where } A = \begin{bmatrix} a_{11} & a_{12} \ldots a_{1j} \ldots a_{1n} \\ a_{21} & a_{22} \ldots a_{2j} \ldots a_{2n} \\ \vdots \\ a_{i1} & a_{i2} \ldots a_{ij} \ldots a_{in} \\ \vdots \\ a_{m1} & a_{m2} \ldots a_{mj} \ldots a_{mn} \end{bmatrix}$$

a_{ij}'s are called elements of the matrix A, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

TYPES OF MATRICES

- **Rectangular matrix :** If $m \neq n$, then $m \times n$ matrix is called rectangular matrix.
- **Square matrix :** If $m = n$, then $m \times n$ matrix is called square matrix.
- **Row matrix :** Matrix with only one row is called row matrix.
- **Column matrix :** Matrix with only one column is called column matrix.
- **Diagonal matrix :** Diagonal matrix is a square matrix in which all the elements except the main diagonal elements are zero, *i.e.*, $a_{ij} = 0 \ \forall \ i \neq j$.
  Note : (i) Diagonal matrix of order n is denoted by diag $[a_{11}, a_{22}, \ldots, a_{nn}]$.
  (ii) If $A = \text{diag } [a, b, c]$, then $A^n = \text{diag } [a^n \ b^n \ c^n]$
- **Scalar matrix :** Scalar matrix is a square matrix in which all the diagonal elements are identical and all other elements being zero.
- **Identity or unit matrix :** Identity matrix is a square matrix $[a_{ij}]_{n \times n}$ in which
  (i) $a_{ij} = 0 \ \forall \ i \neq j$ (ii) $a_{ij} = 1 \ \forall \ i = j$
  Note : An identity matrix of order n is denoted by I_n.
- **Null matrix (zero matrix) :** The matrix whose all elements are zero is called null or zero matrix. We denote it by O.
- **Upper triangular matrix :** A square matrix $A = [a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0 \ \forall \ i > j$.
- **Lower triangular matrix :** A square matrix $A = [a_{ij}]$ is called lower triangular matrix if $a_{ij} = 0 \ \forall \ i < j$.
- **Sub matrix :** A matrix obtained by deleting some rows or some columns (or both) of a matrix is called sub matrix.
- **Horizontal matrix :** A matrix in which number of columns are more than the number of rows is called horizontal matrix.
- **Vertical matrix :** A matrix in which the number of rows are more than the number of columns is called vertical matrix.
- **Equal matrices :** Two matrices A and B are equal if and only if both matrices are of same order and element of one matrix is equal to the corresponding element of other matrix. If two matrices A and B are equal, then we write $A = B$.

By : **R. K. Tyagi**, Retd. Principal, HOD Maths, Samarth Shiksha Samiti, New Delhi

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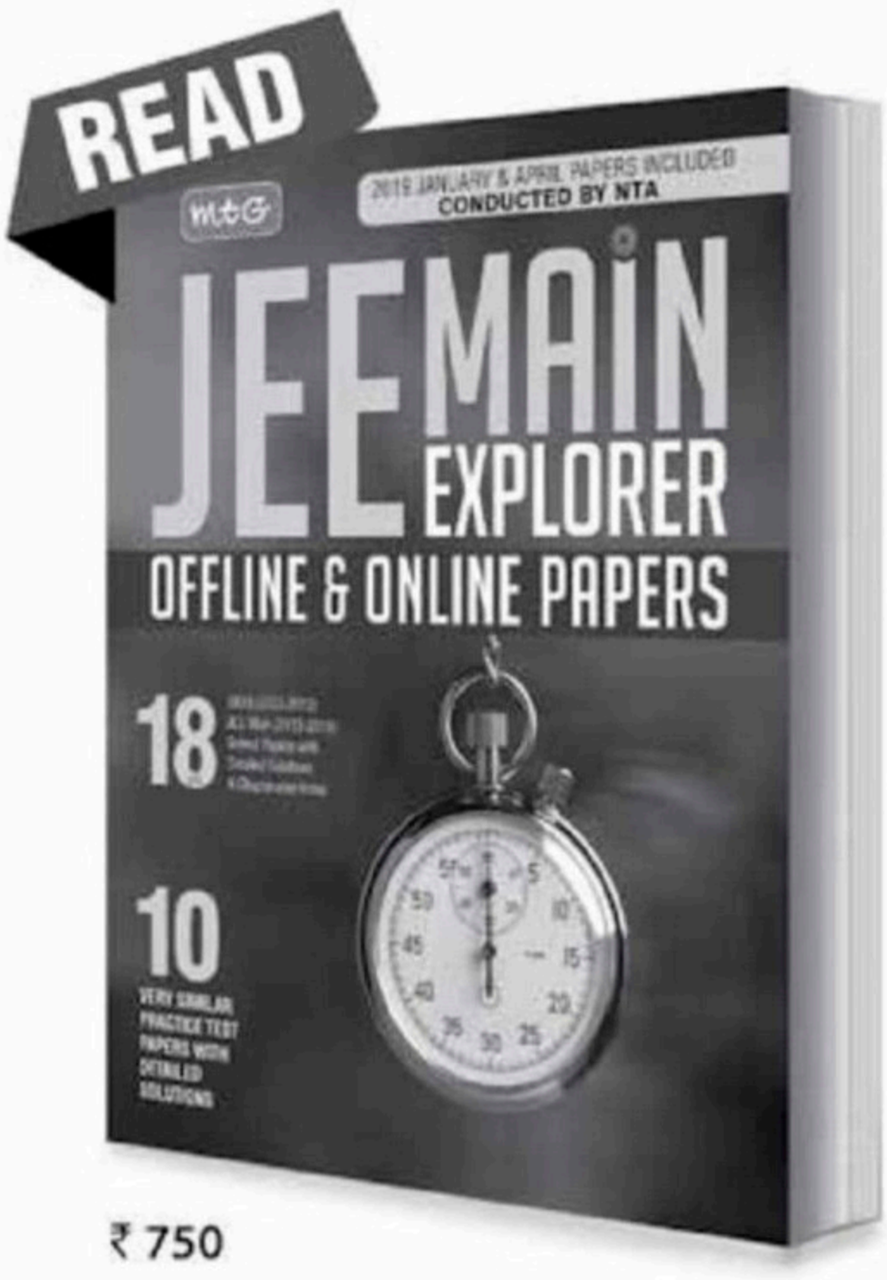

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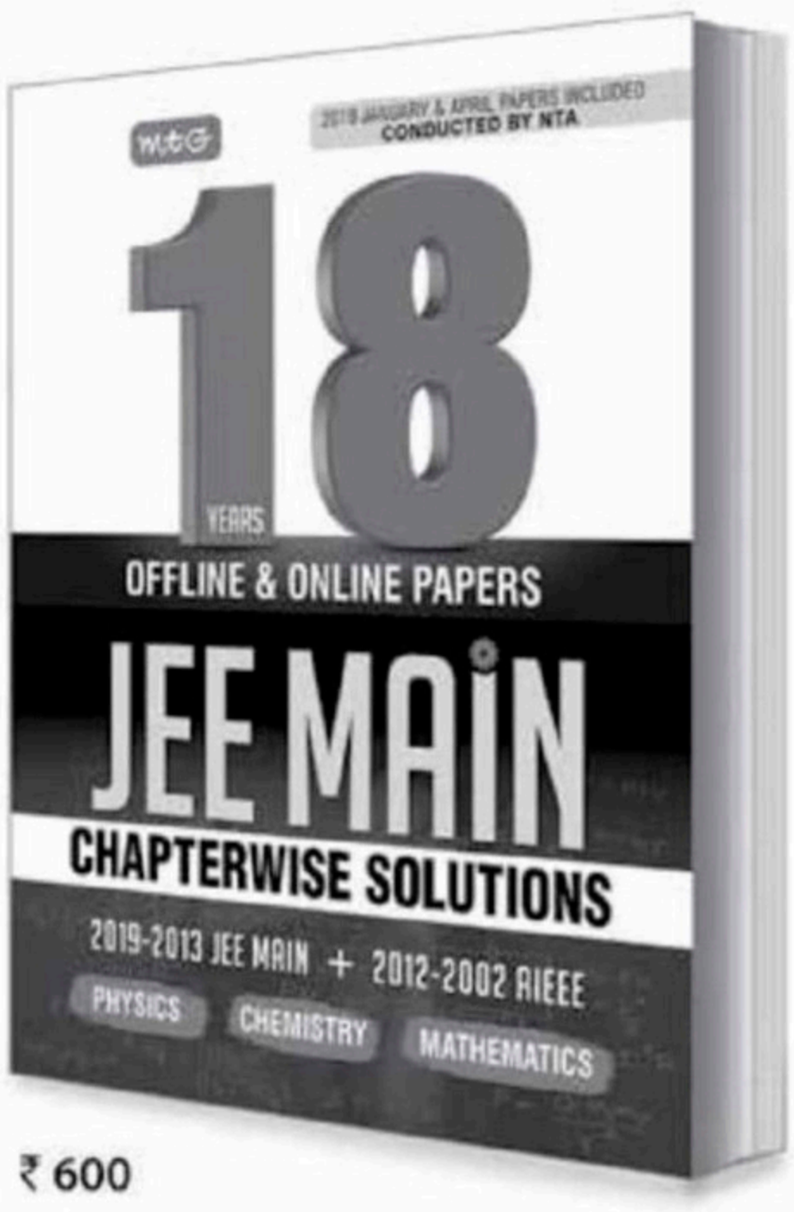

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OPERATIONS ON MATRICES

- **Addition of matrices :** Two matrices A and B can be added only if they have same number of rows and columns.
  If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$, then
  $A + B = [a_{ij} + b_{ij}]_{m \times n}$.
- **Subtraction of matrices :** Two matrices A and B can be subtracted only if they have same number of rows and columns. Subtraction is denoted by $A - B$ or $B - A$.
- **Scalar multiplication :** Let $A = [a_{ij}]_{m \times n}$ be a matrix and k is a scalar. Thus, $kA = [ka_{ij}]_{m \times n}$.
- **Multiplication of matrices :** Let A be a matrix of order $m \times n$ and B of order $n \times p$, then the product AB is defined to be a matrix of order $m \times p$, such that its element in i^{th} row and j^{th} column is the sum of products of corresponding elements of the i^{th} row of A and j^{th} column of B.

Properties of Matrix Addition

If A, B and C are matrices of the same order, then

- $A + B = B + A$ (commutative property of addition)
- $A + (B + C) = (A + B) + C$ (associative property of addition)
- $A + O = O + A = A$ (O is additive identity)
- $A + (-A) = (-A) + A = O$ (additive inverse)
- $A + B = A + C \Rightarrow B = C$ (left cancellation law)
- $B + A = C + A \Rightarrow B = C$ (right cancellation law)

Properties of Matrix Multiplication

- For two matrices A and B, the matrix multiplication is not commutative in general, *i.e.*, $AB \neq BA$.
- If A and B are diagonal matrices of same order, then AB and BA will always be equal. *i.e.*, $AB = BA$.
- The product of two matrices can be a zero matrix while none of them is a zero matrix.
- If $AB = O$, then it is not necessarily implies $A = O$ or $B = O$ or $BA = O$.
- Matrix multiplication is associative, *i.e.*, $(AB)C = A(BC)$
- Matrix multiplication is distributive over addition *i.e.*, $A(B + C) = AB + AC$
- For every square matrix A of order $m \times n$ there exist a left multiplicative identity denoted by I_m and a right multiplicative identity denoted by I_n such that $I_m A = A = AI_n$.

Properties of Scalar Multiplication

Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ are two matrices and k_1 and k_2 are two scalars, then

- $k_1(A + B) = k_1A + k_1B$
- $(k_1 + k_2)A = k_1A + k_2A$
- $k_1k_2A = k_1(k_2A) = k_2(k_1A) = k_2k_1A$
- $(-k)A = -(kA) = k(-A)$
- $IA = AI = A$

TRANSPOSE OF A MATRIX

Let A be a $m \times n$ matrix, then the $n \times m$ matrix, obtained by interchanging the rows and columns of A, is called the transpose of A and is denoted by A^T or A'.
If $A = [a_{ij}]_{m \times n}$, then $A^T = [a_{ji}]_{n \times m}$

Properties of Transpose of a Matrix

- $(A^T)^T = A$
- $(A + B)^T = (B + A)^T = B^T + A^T = A^T + B^T$ (commutative law of addition)
- $(kA)^T = kA^T$, k be any scalar.
- $(AB)^T = B^TA^T$, provided product AB is defined.
- $(A_1 A_2 A_3 \ldots A_n)^T = A_n^T {A_{n-1}}^T \ldots A_2^T A_1^T$, provided product $A_1A_2 \ldots\ldots A_n$ is defined.

TRACE OF A MATRIX

The sum of the main diagonal elements of a square matrix A is called the trace of A and it is denoted by $t_r(A)$.

$$i.e., t_r(A) = \sum_{i=1}^{m} a_{ii} = a_{11} + a_{22} + \ldots + a_{mm}$$

Properties of Trace of a Matrix

- If A and B are of same order matrices, then $t_r(A + B) = t_r(A) + t_r(B)$
- If A and B are of order $n \times m$ and $m \times n$ respectively, then $t_r(AB) = t_r(BA)$.
- $t_r(A) = t_r(A^T)$, where A^T is transpose of A.

SYMMETRIC AND SKEW SYMMETRIC MATRICES

- **Symmetric Matrix :** A square matrix $A = [a_{ij}]$ is said to be symmetric matrix iff $a_{ij} = a_{ji}$ for all i, j, *i.e.*, iff $A^T = A$.
- **Skew Symmetric Matrix :** A square matrix $A = [a_{ij}]$ is said to be a skew symmetric matrix iff $a_{ij} = -a_{ji}$ for all i, j, *i.e.*, iff $A^T = -A$.
  Note : For a skew symmetric matrix A, $a_{ij} = -a_{ji}$ $\forall$ $i = j$, *i.e.*, every diagonal elements of A is zero.

Some Results on Symmetric and Skew Symmetric Matrices

- The sum of two symmetric matrices is symmetric.
- The sum of two skew symmetric matrices is skew symmetric.
- Let A be a square matrix and k be a scalar, then
  (i) if A is symmetric, then kA is also symmetric.
  (ii) if A is skew symmetric, then kA is also skew symmetric.
- For any square matrix A,
  (i) $A + A^T$ is symmetric.

(ii) $A - A^T$ is skew symmetric.
(iii) AA^T and A^TA are symmetric.

- Every square matrix can be uniquely expressed as the sum of a symmetric and a skew symmetric matrix.

$$i.e.,\quad A=\frac{1}{2}(A+A^T)+\frac{1}{2}(A-A^T)=P+Q \text{ (say)},$$

where P and Q are respectively symmetric and skew symmetric parts of A.

- If A and B are symmetric matrices, then
(i) $AB + BA$ is symmetric matrix.
(ii) $AB - BA$ is skew symmetric matrix.

SPECIAL KIND OF MATRICES

- **Equivalent matrices :** Two matrices A and B are said to be equivalent if one can be obtained from the other by using elementary row transformations and we write $A \sim B$.
- **Idempotent matrix :** A square matrix A is said to be idempotent if $A^2 = A$. Also, if $AB = A$ and $BA = B$, then A and B are said to be idempotent.
- **Orthogonal matrix :** A square matrix A is said to be orthogonal, if $AA^T = A^TA = I$, where I is an identity matrix.
- **Periodic matrix :** A square matrix A is called periodic matrix, if $A^{k+1} = A \ \forall\ k \in N$, the least value of k is called the period of A.
Note : For $k = 1$, we get $A^2 = A$, then periodic matrix is said to be idempotent matrix.
- **Nilpotent matrix :** A square matrix A is called a nilpotent matrix, if it satisfies the condition $A^k = O$ and $A^{k-1} \neq O \ \forall\ k \in N$, the least value of k for which the above conditions hold is called the index or order of nilpotent matrix.
- **Involutory matrix :** A square matrix A is called involutory matrix, if it satisfies the condition $A^2 = I$. For an involutory matrix $(I - A)(I + A) = O$ (null matrix).

POLYNOMIALS IN MATRICES

- Let $f(x) = a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n$, where a_i $(i = 0, 1, 2, ..., n)$ are all constants and $n \in N$, then $f(x)$ is called polynomial of degree n provided $a_0 \neq 0$. If the variable x replaced by matrix A, then $f(A) = a_0A^n + a_1A^{n-1} + ... + a_{n-1}A + a_nI$ is called a matrix polynomial.

DETERMINANTS

To every square matrix $A = [a_{ij}]$ of order m, we can associate a number (real or complex) called determinant of the square matrix A denoted by det A or $|A|$.

MINORS AND CO-FACTORS OF A MATRIX

Let $A=\begin{bmatrix} a_{11} & b_{12} & c_{13} \\ a_{21} & b_{22} & c_{23} \\ a_{31} & b_{32} & c_{33} \end{bmatrix}$ *i.e.*, $A=[a_{ij}]_{3\times3}$,

then determinant obtained by deleting the i^{th} row and j^{th} column is called the minor of the element exist at i^{th} row and j^{th} column. We denote it by M_{ij} and the co-factor of the same element is $(-1)^{i+j} M_{ij}$, which we denote by C_{ij}. The determinant of the matrix A is

$$|A|=\begin{vmatrix} a_{11} & b_{12} & c_{13} \\ a_{21} & b_{22} & c_{23} \\ a_{31} & b_{32} & c_{33} \end{vmatrix} = \begin{matrix} a_{11}C_{11}+b_{12}C_{12}+c_{13}C_{13} \\ \text{or} \\ a_{11}C_{11}+a_{21}C_{21}+a_{31}C_{31} \end{matrix}$$

$|A|$ can be obtained by expanding it along any row or column.

PROPERTIES OF DETERMINANTS

- If rows be changed into columns and vice versa, then the value of determinant remains unaltered.
- Let $A = [a_{ij}]_{m \times m}$ then $|A| = 0$ iff
(i) Any two rows or columns are identical.
(ii) Any two rows or columns are proportional.
(iii) Each element of any row or any column is zero.
- If any two rows (columns) of a determinant are interchanged, then the resulting determinant changes its sign.
- If any line (row/column) of a determinant D be passed over n parallel lines (row/column), then the resulting determinant D' is equal to $(-1)^nD$.

i.e., let $D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and D'' obtained after passing over 2 parallel lines (column) *i.e.*,

$$D''=\begin{vmatrix} b_1 & c_1 & a_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix} = (-1)^2 D.$$

- If each element of any row (or column) of a determinant be multiplied by a non-zero constant k, then the value of the determinant get raised by k times.
- If each element of any row (or column) is the sum of two (or more) terms, then the determinant can be expressed as the sum of two (or more) determinants of same order in the following way :

$$\begin{vmatrix} k_1+a_1 & b_1 & c_1 \\ k_2+a_2 & b_2 & c_2 \\ k_3+a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

- If each element of a line (row or column) of a determinant be added to the equimultiples of the corresponding elements of one or more parallel lines (row/column), then the value of the determinant remains unaltered.
- If a determinant D vanishes for $x = a$, then $(x - a)$ is a factor of D.

SPECIAL OPERATIONS ON DETERMINANTS

- **Product of two determinants :** The product of two determinants can be obtained by multiplying row by column, row by row, column by column or column by row.

Let $D_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ and $D_2 = \begin{vmatrix} l_1 & l_2 \\ m_1 & m_2 \end{vmatrix}$, then

$$D_1 \times D_2 \text{ (row by column)} = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} l_1 & l_2 \\ m_1 & m_2 \end{vmatrix}$$

$$= \begin{vmatrix} a_1l_1 + b_1m_1 & a_1l_2 + b_1m_2 \\ a_2l_1 + b_2m_1 & a_2l_2 + b_2m_2 \end{vmatrix}$$

and other three similarly.

- **Differentiation of a determinant**

Let $D(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix}$, then

$$D'(x) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l'(x) & m'(x) & n'(x) \\ u(x) & v(x) & w(x) \end{vmatrix}$$

$$+ \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u'(x) & v'(x) & w'(x) \end{vmatrix}$$

- **Limit of a determinant :** Limit on a determinant is applied only if limit of each element of each row (column) is exist finitely at a given point (say $x = a$)

Let $D(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix}$, then

$$\lim_{x \to a} D(x) = \begin{vmatrix} \lim\limits_{x \to a} f(x) & \lim\limits_{x \to a} g(x) & \lim\limits_{x \to a} h(x) \\ \lim\limits_{x \to a} l(x) & \lim\limits_{x \to a} m(x) & \lim\limits_{x \to a} n(x) \\ \lim\limits_{x \to a} u(x) & \lim\limits_{x \to a} v(x) & \lim\limits_{x \to a} w(x) \end{vmatrix}$$

- **Integration of a determinant**

Let $D(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ u & v & w \end{vmatrix}$, then

$$\int D(x)\,dx = \begin{vmatrix} \int f(x)\,dx & \int g(x)\,dx & \int h(x)\,dx \\ l & m & n \\ u & v & w \end{vmatrix}$$

Note :

(i) If limits of integration is given, then integrate each element (variable quantity) of first row within the limits.

(ii) If more than one row have variable quantity, then first expand (fixed) determinant of $D(x)$ *i.e.*, find $|D(x)|$, then integrate within the given limits.

(iii) If each row (or column) of $D(x)$ has variable quantity, then evaluate the integration as like in differentiation of a determinant.

ADJOINT OF A SQUARE MATRIX

Let $A = [a_{ij}]$ be a square matrix of order n. Let C_{ij} be co-factor of a_{ij}. Then, the transpose of matrix of co-factors of elements of A is called adjoint of A. It is denoted by adj$A$ and given by adj $A = [C_{ij}]^T$.

Properties of Adjoint of a Matrix

If A and B are two square matrices of order n and I_n be the corresponding identity (unit) matrix, then

- adj (AB) = (adj B) (adj A)
- A(adj A) = $|A|I_n$ = (adj A) A
- $|\text{adj } A| = |A|^{n-1}$ if $|A| \neq 0$
- adj(adj A) = $|A|^{n-2} \cdot A$
- If $|A| = 0$, then (A adj A) = A(adj A) = (adj A) $A = 0$
- $|\text{adj (adj } A)| = |A|^{(n-1)^2}$
- adj$(I_n) = I_n$
- adj$(kA) = k^{n-1}$ (adj A), k is scalar
- adj $A = O$ if A is null matrix.
- adj (A^m) = (adj $A)^m$, where m is positive integral power of A.

Note : (i) If A is a symmetric matrix, then adj A is also a symmetric.

(ii) If A is diagonal matrix, then adj A is also a diagonal matrix.

(iii) If A is triangular matrix, then adj A is also a triangular matrix.

SINGULAR AND NON-SINGULAR MATRICES

A square matrix $A = [a_{ij}]_{m \times m}$ is said to be singular matrix if its determinant value is zero, otherwise it is called non-singular.

- If A is singular, then $|\text{adj } A| = 0$.
- If $A^T = -A$ (*i.e.*, A is skew symmetric) and if order of A is odd, then $|A| = 0$.

INVERSE OF A MATRIX

If A and B are two square matrices of same order such that $AB = BA = I$, then B is called inverse of matrix A. It is denoted by A^{-1}. If $AB = BA = I$, then A and B are inverse of each other. *i.e.*, $A^{-1} = B$ and $B^{-1} = A$.

- A square matrix A is said to be invertible (*i.e.*, A^{-1} exist) iff it is non-singular.
- If $A = [a_{ij}]_{m \times n}$ is any square matrix, then its inverse is given by

$$A^{-1} = \frac{1}{|A|}(\text{adj } A), \text{ provided } |A| \neq 0$$

Properties of Inverse of a Matrix

- $(A^{-1})^{-1} = A$
- $(A^T)^{-1} = (A^{-1})^T$
- $AA^{-1} = A^{-1}A = I$
- $(A^n)^{-1} = (A^{-1})^n, n \in N$
- $\text{adj}(A^{-1}) = (\text{adj } A)^{-1}$
- $(AB)^{-1} = B^{-1}A^{-1}$, where $|A| \neq 0, |B| \neq 0$
- For three non-singular matrices A, B and C we have $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
- $(A_1 A_2 A_3 \ldots A_{n-1} A_n)^{-1} = A_n^{-1}A_{n-1}^{-1} \ldots A_2^{-1} A_1^{-1}$ provided each A_i, $i = 1, 2, \ldots, n$, $|A_i| \neq 0$.
- Let $A = \text{diag}(a_1\ a_2 \ldots a_n)$ such that $|\text{diag } (a_1\ a_2 \ldots a_n)| \neq 0$, then
  (i) A^{-1} is also diagonal matrix.
  (ii) Every diagonal matrix is a symmetric and inverse of $\text{diag}(a_1\ a_2 \ldots a_n)$ matrix is also symmetric.

CHARACTERISTIC EQUATION OF A MATRIX

- Let $A = [a_{ij}]_m$ be a square matrix, then matrix $A - \lambda I$ is called the characteristic matrix of A and $|A - \lambda I| = 0$ is called characteristic equation of A and its roots are called characteristic roots or latent roots or eigen values.

r-ROWED MINOR OF A MATRIX

- Let $A = [a_{ij}]_{m \times n}$ be a matrix. If any r row and any r column from A are retained and remaining $(m - r)$ rows and $(n - r)$ columns are removed, then the determinant of the remaining $r \times r$ sub matrix of A is called a minor of A of order r (or r-rowed minor of A).

RANK OF A MATRIX

The rank of a matrix A is called 'r' if

- It has atleast one non-zero minor of order 'r'.
- Every minor of A of order higher than 'r' is zero.
- The rank r of matrix A is denoted by $\rho(A)$.

Remarks :

(i) If there exist a non-zero minor of order 'r' then rank of matrix A is $\geq r$.

(ii) If all minors of order $(r + 1)$ is zero, then rank of matrix A is $\leq r$.

(iii) Rank of every null matrix is always zero.

(iv) Rank of an identity or diagonal matrices is equal to the order of the matrix.

(v) Rank of a matrix whose each element is unity is 1.

METHODS OF SOLVING THE SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

Cramer's Rule

Consider the system of simultaneous linear equations in three variables.

$a_1x + b_1y + c_1z = d_1$; $a_2x + b_2y + c_2z = d_2$ and $a_3x + b_3y + c_3z = d_3$

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$,

$D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$, $D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

- If $D \neq 0$, then the system of equations is consistent and has a unique solution and solution is given by

$$x = \frac{D_1}{D};\ y = \frac{D_2}{D} \text{ and } z = \frac{D_3}{D}$$

- If $D = 0 = D_1 = D_2 = D_3$, then system of equations is consistent and has infinitely many solutions and general solution can be generated by putting $z = k$, and solving any two equations for values of x and y in terms of k $(k \neq 0)$.

EXAM ALERT

Kerala Engineering Entrance Examination Dates Announced 2020-21

The Entrance Examination for admission to Engineering courses in Kerala, for the year 2020-21 will be conducted as per the following schedule:

Dates and Time of Engineering Entrance Examination		
Subject	**Date**	**Time**
Physics and Chemistry	20th April	10:00 AM - 12:30 PM
Mathematics	21st April	10:00 AM - 12:30 PM

Candidates who desire to get admission in the various Engineering Courses in Kerala shall have to appear and qualify in the Kerala Engineering Entrance Examination to be conducted by the Commissioner for Entrance Examinations, Kerala.

- If $D = 0$ and at least one of the determinants (D_1 or D_2 or D_3) is non-zero, then system of equations is inconsistent and have no solution.

Matrix Method to Solve the System of Non-Homogeneous Simultaneous Equations of the form $AX = B$.

- If $|A| \neq 0$, then system $AX = B$ has a unique solution and given by $X = A^{-1} B$.
- If $|A| \neq 0$ and (adj A) $(B) \neq O$, then system of equations is inconsistent and has no solution.
- If $|A| \neq 0$ and (adj A) $(B) = O$, then system of equations is consistent and has infinitely many solution.

System of Linear Homogeneous Simultaneous Equations of the form $AX = O$.

- If $|A| \neq 0$, then system of equations is consistent with unique solution namely $x = y = z = 0$ known as trivial solution.
- If $|A| = 0$, then system of equations is also consistent with infinitely many solutions and general solution can be generated by putting $z = k$ *(any real number)* and the values of x and y can be obtained in terms of k.

PROBLEMS

Single Option Correct Type

1. If $f(x) = \begin{vmatrix} x^3 & \cos x & P^{x^4} \\ \sin x & x^4 & \sec x \\ \tan x & 2 & 3 \end{vmatrix}$, then value of $\int_{-\pi/4}^{\pi/4} f(x)dx$ is

(a) $\frac{\pi^2}{16}$ (b) -2 (c) 0 (d) $\frac{\pi}{16}$

2. If $D(x) = \begin{vmatrix} x^2-4x+3 & 2x-4 & 1 \\ 3x^2+x+4 & 6x+1 & 3 \\ 7x^2-6x+5 & 14x-6 & 7 \end{vmatrix}$ $= ax^3 + bx^2 + cx + d$ where $a + b + c + d = \lambda\alpha\lambda$, then the value of $(\alpha + \lambda)$ equals

(a) 0 (b) 3 (c) 6 (d) 9

3. If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$ and $C = ABA^T$, then $A^T C^n A$ is equal to, $(n \in N)$

(a) $\begin{pmatrix} -n & 1 \\ 1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & -n \\ 0 & 1 \end{pmatrix}$

(c) A^n (d) $\begin{pmatrix} 1 & 0 \\ -n & 1 \end{pmatrix}$

4. The solution of system of equations $x + (\sin\alpha)y = 1$ and $(\sin\alpha)\, x + 4y = 2$ satisfy $x \geq \frac{4}{5}$ and $y \leq \frac{1}{2}$, then α satisfies the condition

(a) $\alpha \in \left[\frac{\pi}{4}, \frac{\pi}{3}\right]$ (b) $\alpha \in \left[0, \frac{\pi}{6}\right]$

(c) $\alpha \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (d) none of these

5. If $A = \begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}$ and $A^2 = 6A + kI_2$, then value of k equals

(a) -1 (b) 5 (c) -5 (d) 1

6. If $A = \begin{pmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix} = f(x)$, then A^{-1} equals

(a) $-f(-x)$ (b) $(f(x))^2$ (c) $f(-x)$ (d) $-f(x)$

7. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix}$, then A is

(a) nilpotent (b) idempotent
(c) orthogonal (d) periodic

8. If $A(\alpha, \beta) = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & e^{\beta} \end{pmatrix}$, then $A(\alpha, \beta)^{-1}$ in terms of function of A is

(a) $A(\alpha, -\beta)$ (b) $A(-\alpha, -\beta)$
(c) $A(-\alpha, \beta)$ (d) none of these

9. If each $A_1, A_3, A_5, ..., A_{2n-1}$ is a skew symmetric of same order, then $B = \sum_{r=1}^{n} (2r-1)(A_{2r-1})^{2r-1}$ will be

(a) symmetric (b) skew symmetric
(c) neither symmetric nor skew symmetric
(d) none of these

10. Let $f(x) = \frac{1+x}{1-x}$, if A is a matrix such that $A^3 = O$, then $f(A)$ equals

(a) $I - A + A^2$ (b) $I + A + A^2$
(c) $I + 2A + 2A^2$ (d) $I - A - A^2$

11. Let $A = (a_{ij})$ be a square matrix of order 3. Let $B = (b_{ij})$ such that $b_{ij} = 3^{i+j} a_{ij}$ for $1 \leq i, j \leq 3$. If determinant of A is same as its order, then $|B^T|$ is

(a) 3^9 (b) 3^{11} (c) 3^{13} (d) 3^{15}

More Than One Option(s) Correct Type

12. If $A=\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$, then

(a) $AA^T = A^TA = A^2$ (b) $AA^T = 3A$

(c) A is a singular matrix

(d) $A^3 = 9A$

13. Let $A=\begin{pmatrix}1&3&3\\3&1&3\\3&3&1\end{pmatrix}$, then

(a) $A^2 - 5A - 14I_3 = O$

(b) $A^{-1}=\frac{1}{14}(A-5I_3)$

(c) A^2 is invertible

(d) A is not an orthogonal matrix

14. System of equations $x + 3y + 2z = 6$, $x + \lambda y + 2z = 7$ and $x + 3y + 2z = \mu$ has

(a) unique solution for $\lambda = 2$, $\mu \neq 6$

(b) infinitely many solutions for $\lambda = 4$, $\mu = 6$

(c) for $\lambda = 5$, $\mu = 7$ system of equations have no solution

(d) for $\lambda = 5$, $\mu = 3$ system of equations is inconsistent

15. If $AB = A$ and $BA = B$, then

(a) A is idempotent (b) B is idempotent

(c) A^T is idempotent (d) B^T is idempotent

16. Let $|A| \neq 0$, $|B| \neq 0$ and $A^T = A$, $B^T = B$, then which of the following is /are not correct?

(a) B^TAB is symmetric if and only if $A^T = A$

(b) B^TAB is symmetric if and only if $B^T = B$

(c) B^TAB is skew symmetric for every matrix A

(d) B^TAB is skew symmetric if A is skew symmetric

17. Let $A=\begin{pmatrix}l^2+t & lm & ln\\ lm & m^2+t & mn\\ ln & mn & n^2+t\end{pmatrix}$, then det A is divisible by

(a) $l + m^2 + n^2 + t$ (b) $l^2 + m^2 + n^2 + t$

(c) $l^2 + m^2 + n + t^2$ (d) t

18. Find the rank of the matrix

$$A=\begin{pmatrix}1 & 4 & 5\\ \alpha & 8 & 8\alpha-6\\ 1+\alpha^2 & 8\alpha+4 & 2\alpha+21\end{pmatrix}.$$

(a) for $\alpha = 2$, $\rho(A) = 1$ (b) for $\alpha = -1$, $\rho(A) = 2$

(c) for $\alpha \neq -1, 2$, $\rho(A) = 3$

(d) none of these

Comprehension Type

Paragraph for Q. No. 19 to 20

Let $D \neq 0$ and D^c = determinant of matrix of cofactors of D, then $D^c = D^{n-1}$, where $n > 0$ is the order of D. On the basis of above information, answer the following questions :

19. If a, b, c are the roots of the equation $x^3 - px^2 + q = 0$, then the value of $\begin{vmatrix}bc-a^2 & ac-b^2 & ab-c^2\\ ac-b^2 & ab-c^2 & bc-a^2\\ ab-c^2 & bc-a^2 & ac-b^2\end{vmatrix}$ is

(a) p^2 (b) p^4 (c) p^6 (d) p^{10}

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

						8							5						
6	4	7					3					2	9		3	6			4
	2		3				5	4			6			4		8			7
							2								2	9	8	7	
		3				4		9			9		2		4	7	5		3
7			9		5	3							6					4	9
		2	1		8					1		2	9	4			3			5
		1		7					9			1	3	7	2			4		
4	8		6			1		7								7				
						3	2			9	8		7	6						
						4		1		7			2							
						6							8							
	4				7		3					7	4	2	9	6				
	9		5				1	4		6	9	8			4			1		9
		7	1	9		8			7				1		3					
	5			7		6								7		1			8	6
7		2					9						3			7	8			
	6		3		8							1	6		2			4		
5	1		7					3					7	1					2	
8	7		4	5								3		6			2		9	5
		9	8	6			7					9	2			3				

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

20. Let a, b, c are real numbers such that $a + b + c$ is positive and $A = bc - a^2$, $B = ac - b^2$, $C = ab - c^2$ also $\begin{vmatrix} A & B & C \\ B & C & A \\ C & A & B \end{vmatrix} = 81$, then value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ equals

(a) 6561 (b) 9 (c) −6561 (d) −9

Matrix Match Type

21. If A, B, C are square matrices of order 2 with entries of real numbers and $*$ defined as follows, then match the following :

Column-I		Column-II	
P.	If $A*B = \frac{1}{2}(AB + BA)$, then	1.	$A*B = B*A$
Q.	If $A*B = \frac{1}{2}(AB^T + A^TB)$, then	2.	$A*(B + C) = A*B + A*C$
R.	If $A*B = \frac{1}{2}(AB - BA)$, then	3.	$A*I = A$
		4.	$A*A = A^2$
		5.	$A*I = O$

Numerical Value Type

22. If A, B and C are square matrices of order n such that $\det(A) = 3$, $\det(B) = 4$, $\det(C) = 5$, then the value of $[\det(A^2BC^{-1})]$ equals (where $[\cdot]$ represent greatest integral function)

23. The sum of the roots of characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{pmatrix}$ is

24. The rank of the matrix $A = \begin{pmatrix} 1 & -3 & 4 & 7 \\ 9 & 1 & 2 & 0 \end{pmatrix}$ is

25. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and $\vec{A} = (1, a, a^2)$, $\vec{B} = (1, b, b^2), \vec{C} = (1, c, c^2)$ are non coplanar vectors, then absolute value of abc equals

26. If $A = \begin{pmatrix} 1 & 3 \\ 3 & 4 \end{pmatrix}$ and $A^2 - \lambda A - 5I_2 = O_2$, then 3^λ equals

SOLUTIONS

1. (c) : $\because \quad f(x) = \begin{vmatrix} x^3 & \cos x & P^{x^4} \\ \sin x & x^4 & \sec x \\ \tan x & 2 & 3 \end{vmatrix} = -f(-x)$

$\therefore$ $f(x)$ is an odd function, so $\int_{-\pi/4}^{\pi/4} f(x) = 0$

2. (d) : $\because \quad D(x) = \begin{vmatrix} x^2-4x+3 & 2x-4 & 1 \\ 3x^2+x+4 & 6x+1 & 3 \\ 7x^2-6x+5 & 14x-6 & 7 \end{vmatrix}$

$\therefore \quad D'(x) = \begin{vmatrix} 2x-4 & 2x-4 & 1 \\ 6x+1 & 6x+1 & 3 \\ 14x-6 & 14x-6 & 7 \end{vmatrix} + \begin{vmatrix} x^2-4x+3 & 2 & 1 \\ 3x^2+x+4 & 6 & 3 \\ 7x^2-6x+5 & 14 & 7 \end{vmatrix}$

$+ \begin{vmatrix} x^2-4x+3 & 2x-4 & 0 \\ 3x^2+x+4 & 6x+1 & 0 \\ 7x^2-6x+5 & 14x-6 & 0 \end{vmatrix}$

$\Rightarrow \quad D'(x) = 0 \quad (\because C_1 = C_2$ in first determinant and $C_2 = 2C_3$ in second determinant)

$\Rightarrow \quad D(x) =$ constant quantity

$\Rightarrow \quad a = b = c = 0$

$\therefore \quad D(x) = \begin{vmatrix} x^2-4x+3 & 2x-4 & 1 \\ 3x^2+x+4 & 6x+1 & 3 \\ 7x^2-6x+5 & 14x-6 & 7 \end{vmatrix} = 0+0+0+d$

$\Rightarrow \quad D(0) = \begin{vmatrix} 3 & -4 & 1 \\ 4 & 1 & 3 \\ 5 & -6 & 7 \end{vmatrix} = d$

$\Rightarrow \quad d = 98 = 2 \times 7^2 = \lambda\alpha^\lambda \Rightarrow \alpha = 7$ and $\lambda = 2$

$\therefore \quad \alpha + \lambda = 9$

3. (d) : $\because A = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \Rightarrow AA^T = A^TA = I$ ...(i)

Now, $C = ABA^T \Rightarrow A^TC = A^TABA^T = BA^T$ [Using (i)] ...(ii)

Now, $A^TC^nA = A^TCC^{n-1}A = BA^TC^{n-1}A$ [Using (ii)]

$= BA^TCC^{n-2}A = B^2A^TCC^{n-3}A = B^3A^TCC^{n-4}A$

$= ..... = B^{n-1}A^TCA = B^{n-1}BA^TA = B^n$

But $B = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}, B^2 = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}, ..., B^n = \begin{pmatrix} 1 & 0 \\ -n & 1 \end{pmatrix}$

4. (b) : From the given system of equations, we have

$$\begin{pmatrix} \sin\alpha & 4 \\ 1 & \sin\alpha \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \sin\alpha & 4 \\ 1 & \sin\alpha \end{pmatrix}^{-1}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\sin^2\alpha - 4}\begin{pmatrix} \sin\alpha & -4 \\ -1 & \sin\alpha \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\sin^2\alpha - 4}\begin{pmatrix} 2\sin\alpha - 4 \\ \sin\alpha - 2 \end{pmatrix} = \begin{pmatrix} \frac{2}{2+\sin\alpha} \\ \frac{1}{2+\sin\alpha} \end{pmatrix}$$

$$\Rightarrow \quad x = \frac{2}{2+\sin\alpha} \geq \frac{4}{5} \text{ and } y = \frac{1}{2+\sin\alpha} \leq \frac{1}{2}$$

$$\Rightarrow \quad \sin\alpha \leq \frac{1}{2} \text{ and } 0 \leq \sin\alpha \Rightarrow 0 \leq \sin\alpha \leq \frac{1}{2}$$

$$\Rightarrow \quad \sin^{-1}(0) \leq \alpha \leq \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow 0 \leq \alpha \leq \frac{\pi}{6}$$

$$\therefore \quad \alpha \in \left[0, \frac{\pi}{6}\right]$$

5. (c) : $\because \quad A = \begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix} \Rightarrow A^2 = \begin{pmatrix} 1 & 0 \\ -6 & 25 \end{pmatrix}$

Now, $A^2 = 6A + kI_2$

$$\Rightarrow \quad \begin{pmatrix} 1 & 0 \\ -6 & 25 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ -6 & 30 \end{pmatrix} + \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = \begin{pmatrix} 6+k & 0 \\ -6 & 30+k \end{pmatrix}$$

$\Rightarrow \quad 6 + k = 1$ or $30 + k = 25 \Rightarrow k = -5$

6. (c) : Here, $|A| = 1$ and $\text{adj } A = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\text{Now, } A^{-1} = \frac{\text{adj } A}{|A|} = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix} = f(-x)$$

7. (a) : We have, $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix}$

$\Rightarrow \quad A^2 \neq A$, ($A$ is not an idempotent matrix)

Also, $AA^T \neq A^TA \neq I$ (A is not an orthogonal matrix)

Also, $A^{k+1} \neq A, k \in N$ (A is not a periodic matrix)

Now, a square matrix A is said be nilpotent if $A^k = O$ and $A^{k-1} \neq O, k \in N$.

$\because \quad A^2 = O$, but $A \neq O \therefore A$ is nilpotent matrix.

8. (b) : $|A(\alpha, \beta)| = (\cos^2\alpha + \sin^2\alpha)e^\beta = e^\beta$

$$\text{Now, } A(\alpha,\beta)^{-1} = \frac{1}{|A(\alpha,\beta)|}\text{adj}\,(A(\alpha,\beta))$$

$$= \frac{\text{adj}\,(A(\alpha,\beta))}{e^\beta} \qquad [\because \ |A(\alpha,\beta)| = e^\beta]$$

$$= \frac{1}{e^\beta}\begin{pmatrix} e^\beta\cos\alpha & -e^\beta\sin\alpha & 0 \\ e^\beta\sin\alpha & e^\beta\cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & e^{-\beta} \end{pmatrix} = A(-\alpha, -\beta)$$

9. (b) : Each of the matrix $A_1, A_3, A_5, ..., A_{2n-1}$ is skew symmetric.

$\Rightarrow \quad A_1{}^T = -A_1, A_3{}^T = -A_3, ..., (A_{2n-1})^T = -A_{2n-1}$

$$\text{Now, } B = \sum_{r=1}^{n}(2r-1)(A_{2r-1})^{2r-1}$$

$B = A_1 + 3A_3{}^3 + 5A_5{}^5 + ... + (2n-1)(A_{2n-1})^{2n-1}$

$\therefore \quad B^T = -(A_1 + 3A_3{}^3 + 5A_5{}^5 + ...$

$+ (2n-1)\,(A_{2n-1})^{2n-1}) = -B$

$\Rightarrow \quad B$ is skew symmetric.

10. (c) : $\because f(x) = \frac{1+x}{1-x} \Rightarrow f(A) = \frac{I+A}{I-A}$

$\Rightarrow \quad (I - A)f(A) = I + A$

$\Rightarrow \quad f(A) = (I - A)^{-1}(I + A) = (I + A + A^2)(I + A)$

$= I + 2A + 2A^2 \ (\because A^3 = O)$

11. (c) : $B = (b_{ij}) = \begin{pmatrix} 3^2a_{11} & 3^3a_{12} & 3^4a_{13} \\ 3^3a_{21} & 3^4a_{22} & 3^5a_{23} \\ 3^4a_{31} & 3^5a_{32} & 3^6a_{33} \end{pmatrix}$

$$\therefore \quad |B^T| = 3^9\begin{vmatrix} a_{11} & 3a_{21} & 3^2a_{31} \\ a_{12} & 3a_{22} & 3^2a_{32} \\ a_{13} & 3a_{23} & 3^2a_{33} \end{vmatrix} = 3^{12}\begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

$= 3^{12}\det(A^T) = 3^{12}\det(A) = 3^{12} \cdot 3 = 3^{13}$

12. (a, b, c, d) : $A^T = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow AA^T = A^TA = A^2$

Now, $AA^T = \begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} = \begin{pmatrix}3&3&3\\3&3&3\\3&3&3\end{pmatrix} = 3A$

Also, $|A| = 0 \Rightarrow A$ is a singular matrix

Again, $A^3 = A^2A = 3AA = 3A^2 = 3(3A) = 9A$

13. (a, b, c) : $A = \begin{pmatrix}1&3&3\\3&1&3\\3&3&1\end{pmatrix} \Rightarrow A^2 = \begin{pmatrix}19&15&15\\15&19&15\\15&15&19\end{pmatrix}$

and $A^T = A$

Now, $AA^T = A^TA = A \cdot A = A^2 = \begin{pmatrix}19&15&15\\15&19&15\\15&15&19\end{pmatrix} \neq I_3$

$\therefore$ A is not an orthogonal matrix.

Again, $A^2 - 5A - 14I_3$

$$= \begin{pmatrix}19&15&15\\15&19&15\\15&15&19\end{pmatrix} - 5\begin{pmatrix}1&3&3\\3&1&3\\3&3&1\end{pmatrix} - 14\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} = O$$

$\therefore$ $A^2 - 5A - 14I_3 = O \Rightarrow A^{-1}A^2 - 5A^{-1}A - 14A^{-1}I_3 = O$

$\Rightarrow$ $(A^{-1}A)A - 5I_3 - 14A^{-1} = O \Rightarrow 14A^{-1} = A - 5I_3$

$$\Rightarrow A^{-1} = \frac{1}{14}(A - 5I_3)$$

Also, $|A^2| = \begin{vmatrix}19&15&15\\15&19&15\\15&15&19\end{vmatrix}$

$= 19(34 \times 4) - 15(15 \times 4) + 15(15 \times (-4))$

$= 2584 - 900 - 900 = 784 \neq 0$ $\therefore$ A^2 is invertible.

14. (b, c, d) : Given equations are

$x + 3y + 2z = 6$ ...(i)

$x + \lambda y + 2z = 7$ ...(ii)

$x + 3y + 2z = \mu$ ...(iii)

Now, $AX = B$

$$\begin{pmatrix}1&3&2\\1&\lambda&2\\1&3&2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}6\\7\\\mu\end{pmatrix}$$

Here, $A = \begin{pmatrix}1&3&2\\1&\lambda&2\\1&3&2\end{pmatrix}$, $X = \begin{pmatrix}x\\y\\z\end{pmatrix}$, $B = \begin{pmatrix}6\\7\\\mu\end{pmatrix}$

$\because$ $|A| = 0$, so, there is no unique solution.

Again system of equations either having no solution if (adj A)$(B) \neq O$ or infinitely many solutions if (adj A)$(B) = O$

$$\text{adj } A = \begin{pmatrix}2\lambda-6&0&6-2\lambda\\0&0&0\\3-\lambda&0&\lambda-3\end{pmatrix}$$

$$\therefore \quad (\text{adj } A)(B) = \begin{pmatrix}2\lambda-6&0&6-2\lambda\\0&0&0\\3-\lambda&0&\lambda-3\end{pmatrix}\begin{pmatrix}6\\7\\\mu\end{pmatrix}$$

If $\lambda = 4$, $\mu = 6$, we get (adj A) $(B) = O$, so infinitely many solutions.

Also, for $\lambda = 5$, $\mu = 7$ and $\lambda = 5$, $\mu = 3$, we get (adj A) $(B) \neq O$, so system of equations have no solution *i.e.*, inconsistent.

15. (a, b, c, d) : (a) $\because AB = A \Rightarrow B(AB) = BA = B$

Now, $(BA)B = B^2$ $\therefore$ $B^2 = B \Rightarrow B$ is idempotent

(b) $BA = B \Rightarrow A(BA) = AB = A$

Now, $(AB)A = A^2$ $\therefore$ $A^2 = A \Rightarrow A$ is idempotent

(c) $(A^T)^2 = A^TA^T = (AA)^T = (A^2)^T = A^T$ ($\because A^2 = A$)

$\therefore$ A^T is idempotent

(d) $(B^T)^2 = B^TB^T = (BB)^T = (B^2)^T = B^T$ ($\because B^2 = B$)

$\therefore$ B^T is idempotent

16. (b, c) : (a) $(B^TAB)^T = B^TA^T(B^T)^T = B^TA^TB = B^TAB$

$\therefore$ B^TAB is symmetric iff $A^T = A$

(b) $(B^TAB)^T = B^TA^T(B^T)^T = B^TA^TB = BAB$, etc.,

$\therefore$ B^TAB is not symmetric iff $B^T = B$

(c) If (B^TAB) is skew symmetric, then $(B^TAB)^T = -B^TAB$

$$\therefore \quad (B^TAB)^T = B^TA^TB = \begin{cases} B^TAB \text{ if } A^T = A \\ -B^TAB \text{ if } A^T = -A \end{cases}$$

but same matrix cannot be symmetric and skew symmetric at the same time.

(d) $(B^TAB)^T = B^TA^TB = B^T(-A)B = -B^TAB$

$= -(B^TA^TB)$

$\Rightarrow$ B^TAB is skew symmetric if A is skew symmetric.

17. (b, d) : Here, $|A| = \begin{vmatrix}l^2+t&lm&ln\\lm&m^2+t&mn\\ln&mn&n^2+t\end{vmatrix}$

$$= \frac{lmn}{lmn}\begin{vmatrix}l^2+t&l^2&l^2\\m^2&m^2+t&m^2\\n^2&n^2&n^2+t\end{vmatrix}$$

[Applying $R_1 \to lR_1$, $R_2 \to mR_2$, $R_3 \to nR_3$ and taking l, m, n common from C_1, C_2 and C_3]

$$= (l^2+m^2+n^2+t)\begin{vmatrix}1&1&1\\m^2&m^2+t&m^2\\n^2&n^2&n^2+t\end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2 + R_3$ and taking common $(l^2 + m^2 + n^2 + t)$ from R_1]

$$= (l^2+m^2+n^2+t)\begin{vmatrix} 0 & 0 & 1 \\ -t & t & m^2 \\ 0 & -t & n^2+t \end{vmatrix} = t^2(l^2+m^2+n^2+t),$$

[Applying $C_1 \to C - C_2$, $C_2 \to C_2 - C_3$]

Which is divisible by t, t^2 and $(l^2 + m^2 + n^2 + t)$.

18. (a, b, c) : Given, $A = \begin{pmatrix} 1 & 4 & 5 \\ \alpha & 8 & 8\alpha-6 \\ 1+\alpha^2 & 8\alpha+4 & 2\alpha+21 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 4 & 5 \\ \alpha-2 & 0 & 8(\alpha-2) \\ \alpha^2 & 8\alpha & 2(\alpha+8) \end{pmatrix}$$

[Applying $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$]

Now, for $\alpha = 2$, we have

$$A = \begin{pmatrix} 1 & 4 & 5 \\ 0 & 0 & 0 \\ 4 & 16 & 20 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

[Applying $R_3 \to R_3 - 4R_1$]

$\therefore \quad \rho(A) = 1$

Again, for $\alpha = -1$, $A = \begin{pmatrix} 1 & 4 & 5 \\ -3 & 0 & -24 \\ 1 & -8 & 14 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 5 \\ 0 & 12 & -9 \\ 0 & -12 & 9 \end{pmatrix}$

[Applying $R_2 \to R_2 + 3R_1$, $R_3 \to R_3 - R_1$]

$$= \begin{pmatrix} 1 & 4 & 5 \\ 0 & 12 & -9 \\ 0 & 0 & 0 \end{pmatrix}$$

[Applying $R_3 \to R_3 + R_2$]

$\therefore \quad \rho(A) = 2$

Also, we can observe that if $\alpha \neq -1, 2$ then $\rho(A) = 3$

19. (c) : $\because$ a, b, c are roots of $x^3 - px^2 + q = 0$

$\therefore \quad a + b + c = p$, $ab + bc + ca = 0$ and $abc = q$ ...(i)

$$\text{Let } D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \Rightarrow D^c = \begin{vmatrix} bc-a^2 & ac-b^2 & ab-c^2 \\ ac-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ac-b^2 \end{vmatrix}$$

where, D^c = det of matrix of co-factors of $D = D^{3-1} = D^2$

$$\text{Now, } D^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$$

$$= \begin{vmatrix} a^2+b^2+c^2 & 0 & 0 \\ 0 & a^2+b^2+c^2 & 0 \\ 0 & 0 & a^2+b^2+c^2 \end{vmatrix}$$

[Using row-column multiplication]

$= (a^2 + b^2 + c^2)^3 = [(a + b + c)^2 - 2(ab + bc + ca)]^3$

[Using (i)]

$= (p^2 - 0)^3 = p^6$

20. (b) : $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} A & B & C \\ B & C & A \\ C & A & B \end{vmatrix} = 81,$

where $A = bc - a^2$, $B = ac - b^2$, $C = ab - c^2$

$$\Rightarrow \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \pm 9 \ \therefore \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 9 \ [\because a + b + c > 0]$$

PUZZLE CORNER

MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

6×	15×		10+		6÷
	24×	11+	4–		
1–			5+		2–
			12×	900×	
8×		18×			

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

21. P → 1, 2, 3, 4 ; Q → 2 ; R → 2, 5

(P) $A*B=\frac{1}{2}(AB+BA)=\frac{1}{2}(BA+AB) = B*A$

Also, $A*(B+C)=\frac{1}{2}\{A(B+C)+(B+C)A\}$

$=\frac{1}{2}(AB+BA)+\frac{1}{2}(AC+CA) = A*B + A*C$

Also, $A*I=\frac{1}{2}(AI+IA)=\frac{1}{2}(A+A)=A$

Again, $A*A=\frac{1}{2}(A\cdot A+A\cdot A)=A^2$

(Q) $A*B=\frac{1}{2}(AB^T+A^TB)=\frac{1}{2}(A^TB+AB^T)$

$\neq\frac{1}{2}(BA^T+B^TA)=B*A$

Also, $A*(B+C)=\frac{1}{2}(A(B+C)^T+A^T(B+C))$

$=\frac{1}{2}(A(B^T+C^T)+A^TB+A^TC)$

$=\frac{1}{2}(AB^T+A^TB)+\frac{1}{2}(AC^T+A^TC) = A*B + A*C$

Also, $A*I=\frac{1}{2}(AI^T+A^TI)=\frac{1}{2}(A+A^T)\neq A$ or O

Again, $A*A=\frac{1}{2}(AA^T+A^TA)\neq A^2$

(R) $A*B=\frac{1}{2}(AB-BA)=-\frac{1}{2}(BA-AB)=-(B*A)$

Also, $A*(B+C)=\frac{1}{2}(A(B+C)-(B+C)A)$

$=\frac{1}{2}(AB+AC-BA-CA)$

$=\frac{1}{2}(AB-BA)+\frac{1}{2}(AC-CA) = A*B + A*C$

Now, $A*I=\frac{1}{2}(AI-IA)=\frac{1}{2}(A-A)=O$

Also, $A*A=\frac{1}{2}(AA-AA)=\frac{1}{2}(A^2-A^2)=O\neq A^2$

22. (7) : Given, $|A| = 3$, $|B| = 4$ and $|C| = 5$

Now, $\det(A^2BC^{-1})=|A^2BC^{-1}|=\frac{|A|^2|B|}{|C|}=\frac{9\times4}{5}$

$\therefore\quad [\det(A^2BC^{-1})]=\left[\frac{36}{5}\right]=7$

23. (4) : The characteristic equation of a matrix is given by $|A - \lambda I| = 0$.

$\Rightarrow \begin{vmatrix}1-\lambda & 2 & 3\\ 0 & -4-\lambda & 2\\ 0 & 0 & 7-\lambda\end{vmatrix}=0$

$\Rightarrow$ $(1 - \lambda)(-4 - \lambda)(7 - \lambda) = 0 \Rightarrow \lambda = 1, -4, 7$

$\therefore$ Sum of the roots = 1 – 4 + 7 = 4

24. (2) : The given matrix possesses non-zero minors of order 2 like $\begin{vmatrix}1 & -3\\ 9 & 1\end{vmatrix}, \begin{vmatrix}-3 & 4\\ 1 & 2\end{vmatrix}, \begin{vmatrix}4 & 7\\ 2 & 0\end{vmatrix}$, etc.

$\therefore$ Rank of $(A) \geq 2$ ... (i)

Now, A does not possesses any 3-rowed minor.

$\therefore$ Rank of $(A) \leq 2$ ... (ii)

From (i) and (ii), rank of $(A) = 2$

25. (1) : $\because \begin{vmatrix}a & a^2 & 1+a^3\\ b & b^2 & 1+b^3\\ c & c^2 & 1+c^3\end{vmatrix}=0$

$\Rightarrow \begin{vmatrix}a & a^2 & 1\\ b & b^2 & 1\\ c & c^2 & 1\end{vmatrix}+\begin{vmatrix}a & a^2 & a^3\\ b & b^2 & b^3\\ c & c^2 & c^3\end{vmatrix}=0$

$\Rightarrow (1+abc)\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}=0\Rightarrow abc=-1$

[$\because$ $\vec{A}, \vec{B}$ and $\vec{C}$ are non coplanar vectors]

26. (243) : $A=\begin{pmatrix}1 & 3\\ 3 & 4\end{pmatrix} \Rightarrow A^2=\begin{pmatrix}10 & 15\\ 15 & 25\end{pmatrix}$

$\therefore\quad A^2 - \lambda A - 5I_2 = O_2$

$\Rightarrow \begin{pmatrix}10 & 15\\ 15 & 25\end{pmatrix}+\begin{pmatrix}-\lambda & -3\lambda\\ -3\lambda & -4\lambda\end{pmatrix}+\begin{pmatrix}-5 & 0\\ 0 & -5\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$

$\Rightarrow \begin{pmatrix}10-\lambda-5 & 15-3\lambda\\ 15-3\lambda & 25-4\lambda-5\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix} \Rightarrow \lambda = 5$

$\therefore\quad 3^\lambda = 3^5 = 243$

EXAM ALERT 2020

Exam	Date
JEE Main I	6th to 11th January
WB JEE	2nd February
JEE Main II	3rd to 9th April
KEAM	20th to 21st April
MHT CET	16th to 20th May
JEE Advanced	17th May

MOCK TEST PAPER 2020

JEE MAIN

with Numerical Value Type Questions

Exam Date : Between 6th to 11th January

Time : 1 hr. 15 min.

1. A person is permitted to select at least one and at most n coins from a collection of $2n + 1$ distinct coins. If the total number of ways in which he can select coins is 255, then n equals

(a) 4 (b) 8 (c) 16 (d) 32

2. If $\left|z-\frac{3}{z}\right|=2$, then the greatest value of $|z|$ is

(a) 1 (b) 2 (c) 3 (d) 4

3. Let $A=\begin{pmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{pmatrix}$. If $|A^2|=25$, then $|\alpha|=$

(a) $\frac{1}{5}$ (b) 5 (c) 5^2 (d) 1

4. If $f(x)=\begin{vmatrix}x & \sin x & \cos x\\ x^2 & \tan x & -x^3\\ 2x & \sin 2x & 5x\end{vmatrix}$, then $\lim\limits_{x\to 0}\frac{f'(x)}{x}=$

(a) 1 (b) 4 (c) 3 (d) –4

5. In a group of 45 persons, 25 drink tea but not coffee, while 32 drink tea. How many persons drink coffee but not tea?

(a) 12 (b) 13 (c) 15 (d) 20

6. For any two real numbers a and b, we define aRb if and only if $\sin^2 a + \cos^2 b = 1$. The relation R is

(a) reflexive but not symmetric
(b) symmetric but not transitive
(c) transitive but not reflexive
(d) an equivalence relation

7. Let $A = \{x \in R : -1 \le x \le 1\} = B$. Then the mapping $f : A \to B$ given by $f(x) = x|x|$ is

(a) injective but not surjective
(b) surjective but not injective
(c) bijective
(d) none of these

8. If the sum of n terms of an A.P. is $cn(n - 1)$ where $n \neq 0$, then the sum of the squares of these terms is

(a) $c^2n^2(n + 1)^2$ (b) $\frac{2}{3}c^2n(n-1)(2n-1)$

(c) $\frac{2}{3}c^2n(n+1)(2n+1)$ (d) $\frac{1}{3}c^2n^2$

9. Lines passing through the point (11, –2) and touching the circle $x^2 + y^2 = 25$ are

(a) $4x + 3y = 38, 7x - 24y = 125$
(b) $3x + 4y = 25, 7x - 24y = 125$
(c) $3x - 4y = 41, 7x + 24y = 125$
(d) $7x - 24y = 125, 4x - 3y = 38$

10. The probability that Ram will alive 30 years hence is $\frac{7}{11}$ and Shyam will be alive is $\frac{7}{10}$. What is the probability that both Ram and Shyam will be dead 30 years hence?

(a) $\frac{1}{11}$ (b) $\frac{12}{110}$

(c) $\frac{2}{121}$ (d) none of these

11. If the inequality $\frac{mx^2+3x+4}{x^2+2x+2}<5$ is satisfied for all $x \in R$, then

(a) $1 < m < 5$ (b) $-1 < m < 5$

(c) $1 < m < 6$ (d) $m<\frac{71}{24}$

12. The range of the value of the term independent of x in the expansion of $\left(x\sin^{-1}\alpha+\frac{\cos^{-1}\alpha}{x}\right)^{10}$, $\alpha\in[-1,1]$ is

(a) [1, 2] (b) (1, 2)

(c) $\left[\frac{{}^{10}C_5\pi^2}{2^{20}},\frac{-{}^{10}C_5\pi^2}{2^5}\right]$ (d) $\left[-\frac{{}^{10}C_5\pi^{10}}{2^5},\frac{{}^{10}C_5\pi^{10}}{2^{20}}\right]$

By : Sankar Ghosh, S.G.M.C, Mob. 09831244397.

13. The value of $\left(\frac{\cos\alpha+\cos\beta}{\sin\alpha-\sin\beta}\right)^n+\left(\frac{\sin\alpha+\sin\beta}{\cos\alpha-\cos\beta}\right)^n$ (where n is an even number) is equal to

(a) 0 (b) $2\tan^n\frac{\alpha-\beta}{2}$

(c) $2\cot^n\frac{\alpha-\beta}{2}$ (d) $2\cot^n\frac{\alpha+\beta}{2}$

14. The equation $\sqrt{3}\sin x+\cos x=4$ has

(a) infinitely many solutions

(b) no solution

(c) two solutions (d) only one solution

15. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$, has a solution for

(a) $|a|\geq\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}\leq|a|<\frac{1}{\sqrt{2}}$

(c) all real values of a (d) $|a|\leq\frac{1}{\sqrt{2}}$

16. A tangent having slope $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ intersects the major and minor axes at A and B. If O is the origin, then the area of the ΔOAB is (in sq. units)

(a) 48 (b) 9 (c) 24 (d) 16

17. A line passing through the point of intersection of $x + y = 4$ and $x - y = 2$ makes an angle $\tan^{-1}\frac{3}{4}$ with the x-axis. It intersects the parabola $y^2 = 4(x - 3)$ at points (x_1, y_1) and (x_2, y_2) respectively. Then $|x_1 - x_2| =$

(a) $\frac{16}{9}$ (b) $\frac{32}{9}$ (c) $\frac{40}{9}$ (d) $\frac{80}{9}$

18. Let the unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ and $|\vec{a}-\vec{b}|$ is less than 1. If $0\leq\theta\leq\pi$, then θ lies in the interval

(a) $\left[0,\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6},\pi\right]$ (b) $\left(0,\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6},\pi\right)$

(c) $\left[\frac{\pi}{6},\frac{\pi}{2}\right]$ (d) $\left[\frac{\pi}{2},\frac{5\pi}{6}\right]$

19. If $\phi(x)=\int\frac{e^{2\tan^{-1}x}(1+x)^2}{1+x^2}dx=xe^{m\tan^{-1}x}+c$, then $m =$

(a) -2 (b) -1 (c) 1 (d) 2

20. The solution of the differential equation $y\frac{dy}{dx}=x\left[\frac{y^2}{x^2}+\frac{\phi(y^2/x^2)}{\phi'(y^2/x^2)}\right]$ is (where c is a constant)

(a) $\phi\left(\frac{y^2}{x^2}\right)=cx$ (b) $x\phi\left(\frac{y^2}{x^2}\right)=c$

(c) $\phi\left(\frac{y^2}{x^2}\right)=cx^2$ (d) $x^2\phi\left(\frac{y^2}{x^2}\right)=c$

NUMERICAL VALUE TYPE

21. The value of $\lim\limits_{x\to0}\frac{\tan[e^2]x^4-\tan[-e^2]x^4}{\sin^4x}$, (where $[\cdot]$ denotes the greatest integer function) is

22. If the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$, $\frac{x-1}{3}=\frac{y-2}{-1}=\frac{z-3}{4}$ and $\frac{x+k}{3}=\frac{y-1}{2}=\frac{z-2}{h}$ are concurrent, then the value of $2k + h$ is

23. If $xy = e - e^y$, then the value of $\log\left\{\left(\frac{d^2y}{dx^2}\right)_{x=0}\right\}$ is

24. A spherical balloon is being inflated at the rate of 35 cm^3/min. Then the rate of increase of surface area of the balloon when its diameter is 14 cm, is

25. If $f(x)=\begin{cases}3[x]-5\frac{|x|}{x}, & x\neq0\\ 2, & x=0\end{cases}$, then $-2\times\int\limits_{-3/2}^{2}f(x)dx$ is equal to (where $[\cdot]$ denotes the greatest integer function)

SOLUTIONS

1. (a) : The total number of ways, the person can select n coins from $2n + 1$ coin is

$${}^{2n+1}C_1+{}^{2n+1}C_2+{}^{2n+1}C_3+\ldots+{}^{2n+1}C_n=255$$

$$\Rightarrow 1+{}^{2n+1}C_1+{}^{2n+1}C_2+{}^{2n+1}C_3+\ldots+{}^{2n+1}C_n=255+1$$

$$\Rightarrow\frac{1}{2}2^{2n+1}=256\Rightarrow2^{2n}=2^8\Rightarrow2n=8\Rightarrow n=4$$

2. (c) : We have, $\left|z-\frac{3}{z}\right|=2$

Now, $|z|=\left|\left(z-\frac{3}{z}\right)+\frac{3}{z}\right|\leq\left|z-\frac{3}{z}\right|+\left|\frac{3}{z}\right|=2+\left|\frac{3}{z}\right|$

$$\Rightarrow|z|-\frac{3}{|z|}\leq2\Rightarrow|z|^2-2|z|-3\leq0$$

$$\Rightarrow(|z|+1)(|z|-3)\leq0\Rightarrow|z|\geq-1\text{ or }|z|\leq3$$

So, the greatest value of $|z|$ is 3.

3. (a) : $\because |A^2|=|A|^2=\begin{vmatrix}5&5\alpha&\alpha\\0&\alpha&5\alpha\\0&0&5\end{vmatrix}^2=[5(5\alpha)]^2=(25\alpha)^2$

$\because\ |A^2| = 25 \Rightarrow (25\alpha)^2 = 25 \Rightarrow \alpha^2 = \frac{1}{25} \Rightarrow |\alpha| = \frac{1}{5}$

4. (d) : Here, $f(x) = \begin{vmatrix} x & \sin x & \cos x \\ x^2 & \tan x & -x^3 \\ 2x & \sin 2x & 5x \end{vmatrix}$

$$\therefore f'(x) = \begin{vmatrix} 1 & \cos x & -\sin x \\ x^2 & \tan x & -x^3 \\ 2x & \sin 2x & 5x \end{vmatrix} + \begin{vmatrix} x & \sin x & \cos x \\ 2x & \sec^2 x & -3x^2 \\ 2x & \sin 2x & 5x \end{vmatrix} + \begin{vmatrix} x & \sin x & \cos x \\ x^2 & \tan x & -x^3 \\ 2 & 2\cos 2x & 5 \end{vmatrix}$$

$$\Rightarrow \frac{f'(x)}{x} = \begin{vmatrix} 1 & \cos x & -\sin x \\ x & \frac{\tan x}{x} & -x^2 \\ 2x & \sin 2x & 5x \end{vmatrix} + \begin{vmatrix} x & \sin x & \cos x \\ 2x & \sec^2 x & -3x^2 \\ 2 & \frac{\sin 2x}{x} & 5 \end{vmatrix} + \begin{vmatrix} x & \sin x & \cos x \\ x & \frac{\tan x}{x} & -x^2 \\ 2 & 2\cos 2x & 5 \end{vmatrix}$$

$$\Rightarrow \lim_{x\to 0} \frac{f'(x)}{x} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 2 & 5 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 2 & 5 \end{vmatrix}$$

$$= 0 + 2(-1) + 2(-1) = -4$$

5. (b) : Let T and C denote the persons who drink tea and coffee respectively.
Given, $n(T \cup C) = 45$, $n(T - C) = 25$, $n(T) = 32$
Now, $n(T - C) = n(T) - n(T \cap C) = 25$
$\Rightarrow\ n(T \cap C) = 7$
Again, $n(T \cup C) = n(T) + n(C) - n(T \cap C) = 45$
$\Rightarrow\ 32 + n(C) - 7 = 45 \Rightarrow n(C) = 20$
$\therefore\ n(C - T) = n(C) - n(C \cap T) = 20 - 7 = 13$

6. (d) : Given, $aRb \Leftrightarrow \sin^2 a + \cos^2 b = 1$.
Now, $\sin^2 a + \cos^2 a = 1$. So, R is reflexive.
Again, let aRb holds, $\therefore\ \sin^2 a + \cos^2 b = 1$
$\Rightarrow\ 1 - \cos^2 a + 1 - \sin^2 b = 1 \Rightarrow \sin^2 b + \cos^2 a = 1$
$\therefore\ R$ is symmetric.
Again, let aRb holds and bRc holds.
$\therefore\ \sin^2 a + \cos^2 b = 1$ and $\sin^2 b + \cos^2 c = 1$.
Now, $\sin^2 a + \cos^2 b + \sin^2 b + \cos^2 c = 1 + 1$
$\Rightarrow\ \sin^2 a + \cos^2 c = 2 - 1 = 1\ \ \therefore\ R$ is transitive.
Hence, R is an equivalence relation.

7. (c) : Let $x_1, x_2 \in R\ \therefore f(x_1) = f(x_2) \Rightarrow x_1|x_1| = x_2|x_2|$
$\therefore\ f$ is one-one.
Again, let $y \in R$ such that $y = f(x) \Rightarrow y = x|x|$
When $-1 \le x \le 1$, then $-1 \le y \le 1$
$\therefore$ Range and co-domain of f are equal. Thus, f is onto.

8. (b) : Let the A.P. be a, $a + d$, ..., $a + (n - 1)d$.
$S_n = \frac{n}{2}[2a + (n-1)d] = cn(n-1)$
$\therefore\ 2a + (n - 1)d = 2c(n - 1) = 0 + (n - 1)2c$
$\therefore\ a = 0,\ d = 2c$
Now, let $S_k = a^2 + (a + d)^2 + ... + [a + (n - 1)d]^2$
$= d^2 + 2^2 d^2 + ... + (n - 1)^2 d^2$
$= \frac{d^2}{6} n(n-1)(2n-1) = \frac{2}{3} c^2 n(n-1)(2n-1)$

9. (b) : Let the equation of the line be $y + 2 = m(x - 11)$
$\Rightarrow\ mx - y - (11m + 2) = 0$ ...(i)
Since, the line (i) touches the circle $x^2 + y^2 = 25$
$\therefore\ \frac{|-(11m+2)|}{\sqrt{m^2+1}} = 5 \Rightarrow m = -\frac{3}{4}, \frac{7}{24}$
Therefore (i) becomes $3x + 4y = 25$ or $7x - 24y = 125$

10. (b) : Let E_1 and E_2 are two events that Ram and Shyam will alive respectively.
Given, $P(E_1) = \frac{7}{11}$ and $P(E_2) = \frac{7}{10}$
$\therefore\ P(E_1^c) = \frac{4}{11}$ and $P(E_2^c) = \frac{3}{10}$
$\therefore\ P(E_1^c \cap E_2^c) = P(E_1^c)P(E_2^c) = \frac{4}{11} \times \frac{3}{10} = \frac{12}{110}$

11. (d) : We have, $\frac{mx^2 + 3x + 4}{x^2 + 2x + 2} < 5$
$\Rightarrow\ mx^2 + 3x + 4 < 5(x^2 + 2x + 2)$
$\Rightarrow\ (m - 5)x^2 - 7x - 6 < 0,\ \forall\ x \in R$
This is possible if $b^2 - 4ac < 0$ *i.e.*, $49 + 24(m - 5) < 0$
$\Rightarrow\ m < \frac{71}{24}$

12. (d) : $T_{r+1} = {}^{10}C_r \cdot (x \sin^{-1}\alpha)^{10-r} \cdot \left(\frac{\cos^{-1}\alpha}{x}\right)^r$
The term will be independent of x if $10 - 2r = 0 \Rightarrow r = 5$
$T_6 = {}^{10}C_5 (\sin^{-1}\alpha)^5 \cdot (\cos^{-1}\alpha)^5 = {}^{10}C_5 \cdot (\sin^{-1}\alpha \cdot \cos^{-1}\alpha)^5$
$f(\alpha) = \sin^{-1}\alpha \cdot \cos^{-1}\alpha$. Put $\sin^{-1}\alpha = t$
$\therefore f(t) = t\left(\frac{\pi}{2} - t\right),\ -\frac{\pi}{2} \le t \le \frac{\pi}{2}$
$\Rightarrow\ f'(t) = \frac{\pi}{2} - 2t = 0 \Rightarrow t = \frac{\pi}{4}$
$f\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16}, f\left(-\frac{\pi}{2}\right) = -\frac{\pi}{2} \cdot \pi = -\frac{\pi^2}{2}, f\left(\frac{\pi}{2}\right) = 0$
So, the required range
$= \left[{}^{10}C_5 \left(-\frac{\pi^2}{2}\right)^5, {}^{10}C_5 \left(\frac{\pi^2}{16}\right)^5 \right]$

$$=\left[-\frac{{}^{10}C_5\cdot\pi^{10}}{2^5},\frac{{}^{10}C_5\cdot\pi^{10}}{2^{20}}\right]$$

13. (c) : Here, $\left(\frac{\cos\alpha+\cos\beta}{\sin\alpha-\sin\beta}\right)^n+\left(\frac{\sin\alpha+\sin\beta}{\cos\alpha-\cos\beta}\right)^n$

$$=\left(\frac{2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}{2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}}\right)^n+\left(\frac{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}{2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}}\right)^n$$

$$=\left(\cot\frac{\alpha-\beta}{2}\right)^n+\left(-\cot\frac{\alpha-\beta}{2}\right)^n$$

$$=\begin{cases}0, & \text{when } n \text{ is odd}\\ 2\cot^n\frac{\alpha-\beta}{2}, & \text{when } n \text{ is even}\end{cases}$$

14. (b) : The given equation is $\sqrt{3}\sin x+\cos x=4$

$\Rightarrow \frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x=2 \Rightarrow \cos\left(x-\frac{\pi}{3}\right)=2$, which is impossible. As we know that $-1\le\cos\left(x-\frac{\pi}{3}\right)\le 1$.

15. (d) : We know that $-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2}\le 2\sin^{-1}a\le\frac{\pi}{2} \Rightarrow -\frac{\pi}{4}\le\sin^{-1}a\le\frac{\pi}{4}$

$\Rightarrow \sin\left(-\frac{\pi}{4}\right)\le a\le\sin\left(\frac{\pi}{4}\right)\Rightarrow -\frac{1}{\sqrt{2}}\le a\le\frac{1}{\sqrt{2}}\Rightarrow|a|\le\frac{1}{\sqrt{2}}$

16. (c) : Any point on the ellipse $\frac{x^2}{(3\sqrt{2})^2}+\frac{y^2}{(4\sqrt{2})^2}=1$ can be taken as $(3\sqrt{2}\cos\theta,4\sqrt{2}\sin\theta)$ and slope of the tangent $=-\frac{32(3\sqrt{2}\cos\theta)}{18(4\sqrt{2}\sin\theta)}=-\frac{4}{3}\cot\theta$

$\because \quad \frac{-4}{3}\cot\theta=\frac{-4}{3}$ [Given]

$\Rightarrow \quad \cot\theta=1 \Rightarrow \theta=\frac{\pi}{4}$

Hence, equation of tangent is $\frac{x}{6}+\frac{y}{8}=1$

Hence, $A\equiv(6,0)$, $B\equiv(0,8)$.

Area of $\Delta OAB=\frac{1}{2}\times 6\times 8=24$ sq.units

17. (b) : Given that the line is passing through the point (3, 1) (the point of intersection of the straight lines $x+y=4$ and $x-y=2$) and its slope is $\frac{3}{4}$.

$\therefore$ The equation of the line is $3x-4y=5$ ...(i)

Now, putting $y=\frac{3x-5}{4}$ in the equation of parabola, we get $\left(\frac{3x-5}{4}\right)^2=4(x-3)\Rightarrow 9x^2-94x+217=0$

Let x_1 and x_2 are the roots of the equation.

$\therefore \quad x_1+x_2=\frac{94}{9}$ and $x_1x_2=\frac{217}{9}$

Now, $(x_1-x_2)^2=\left(\frac{94}{9}\right)^2-4\times\frac{217}{9}=\frac{1024}{81}$

$\Rightarrow \quad |x_1-x_2|=\frac{32}{9}$

18. (a) : Given that, $\vec{a}$ and $\vec{b}$ are unit vectors and angle between $\vec{a}$ and $\vec{b}$ is 2θ and $|\vec{a}-\vec{b}|<1$.

Now, $|\vec{a}-\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}$

$=2(1-\cos2\theta)=4\sin^2\theta$

$\therefore|\vec{a}-\vec{b}|=2|\sin\theta| \Rightarrow |\sin\theta|<\frac{1}{2}\Rightarrow\theta\in\left[0,\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6},\pi\right]$

19. (d) : $\phi(x)=\int\frac{e^{2\tan^{-1}x}(1+x)^2}{1+x^2}dx$

$=\int\frac{e^{2\tan^{-1}x}(1+x^2+2x)}{1+x^2}dx$

$=\int e^{2\tan^{-1}x}dx+\int\frac{2xe^{2\tan^{-1}x}}{1+x^2}dx$

$=xe^{2\tan^{-1}x}-\int 2e^{2\tan^{-1}x}\times\frac{1}{1+x^2}xdx+\int\frac{2xe^{2\tan^{-1}x}}{1+x^2}dx+c$

$=xe^{2\tan^{-1}x}+c \quad \therefore \quad m=2$

20. (c) : $y\frac{dy}{dx}=x\left[\frac{y^2}{x^2}+\frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$

Putting $\frac{y}{x}=v \Rightarrow y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$, we get

$vx\left(v+x\frac{dv}{dx}\right)=x\left[v^2+\frac{\phi(v^2)}{\phi'(v^2)}\right]$

$\Rightarrow \quad v^2+vx\frac{dv}{dx}=v^2+\frac{\phi(v^2)}{\phi'(v^2)} \Rightarrow \quad v\frac{\phi'(v^2)}{\phi(v^2)}dv=\frac{dx}{x}$

Integrating both sides, we get

$\int\frac{dx}{x}=\int\frac{v\phi'(v^2)}{\phi(v^2)}dv$

$\Rightarrow \quad \ln x+\ln k=\frac{1}{2}\ln\{\phi(v^2)\} \Rightarrow \ln kx=\ln\{\phi(v^2)\}^{\frac{1}{2}}$

$$\Rightarrow \quad kx = \{\phi(v^2)\}^{\frac{1}{2}} \Rightarrow \quad cx^2 = \phi\left(\frac{y^2}{x^2}\right)$$

21. (15) : $\lim\limits_{x\to 0} \dfrac{\tan[e^2]x^4 - \tan[-e^2]x^4}{\sin^4 x}$

$$= \lim_{x\to 0} \frac{\tan 7x^4 - \tan(-8)x^4}{\sin^4 x} = \lim_{x\to 0} \frac{\tan 7x^4 + \tan 8x^4}{\sin^4 x}$$

$$= \frac{7\lim\limits_{x\to 0} \dfrac{\tan 7x^4}{7x^4} + 8\lim\limits_{x\to 0} \dfrac{\tan 8x^4}{8x^4}}{\lim\limits_{x\to 0} \dfrac{\sin x^4}{x^4}} = \frac{7+8}{1} = 15$$

22. (3) : Given, $\frac{x}{1} = \frac{y}{2} = \frac{z}{3} = \lambda$ (say) and

$$\frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-3}{4} = \mu \text{ (say)}$$

Thus, the coordinates of a general point on first line are $(\lambda, 2\lambda, 3\lambda)$ and on second line are $(3\mu + 1, -\mu + 2, 4\mu + 3)$.

If the lines intersect, then they have a common point.

So, for some values of λ and μ, we must have

$\lambda = 3\mu + 1, 2\lambda = -\mu + 2$ and $3\lambda = 4\mu + 3$

or $\lambda - 3\mu = 1, 2\lambda + \mu = 2, 3\lambda - 4\mu = 3$

Solving two of these equations we get $\lambda = 1$ and $\mu = 0$

So, the point of intersection of the first two lines is $(1, 2, 3)$. It lies on the third line.

So, $\dfrac{1+k}{3} = \dfrac{2-1}{2} = \dfrac{3-2}{h}$

$$\Rightarrow \quad \frac{1+k}{3} = \frac{1}{2} \Rightarrow k = \frac{1}{2} \text{ and } \frac{1}{2} = \frac{1}{h} \Rightarrow h = 2$$

$\therefore \quad 2k + h = 3$

23. (–2) : We have, $xy = e - e^y$ ...(i)

$$\Rightarrow \quad x\frac{dy}{dx} + y = -e^y \frac{dy}{dx} \qquad \text{...(ii)}$$

$$\Rightarrow \quad x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = -\left(e^y \frac{d^2y}{dx^2} + \frac{dy}{dx} e^y \frac{dy}{dx}\right)$$

$$\Rightarrow \quad x\frac{d^2y}{dx^2} + e^y \frac{d^2y}{dx^2} + 2\frac{dy}{dx} = -e^y \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad (x + e^y)\frac{d^2y}{dx^2} = -\left[2\frac{dy}{dx} + e^y \left(\frac{dy}{dx}\right)^2\right] \qquad \text{...(iii)}$$

From (ii),

$$(x+e^y)\left(\frac{dy}{dx}\right) = -y \Rightarrow \left(\frac{dy}{dx}\right) = \frac{-y}{(x+e^y)} \Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = -\frac{1}{e}$$

[$\because$ From (i), we get $y = 1$ when $x = 0$]

Now (iii) becomes,

$$(0+e)\left(\frac{d^2y}{dx^2}\right)_{x=0} = -\left(-\frac{2}{e} + e \times \frac{1}{e^2}\right) = -\left(-\frac{1}{e}\right) = \frac{1}{e}$$

$$\therefore \quad \left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{e^2} \Rightarrow \log\left[\left(\frac{d^2y}{dx^2}\right)_{x=0}\right] = -2$$

24. (10) : Let v and s respectively be the volume and surface area of the spherical balloon.

$$\therefore \quad v = \frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Again, $s = 4\pi r^2 \Rightarrow \dfrac{ds}{dt} = 8\pi r \dfrac{dr}{dt}$

Now, $\dfrac{\frac{ds}{dt}}{\frac{dv}{dt}} = \dfrac{2}{r} \Rightarrow \dfrac{ds}{dt} = \dfrac{2}{r} \cdot \dfrac{dv}{dt} = \dfrac{2}{7} \times 35 = 10$

Thus, the rate of increase of the surface area is 10 cm^2/min, when diameter of the balloon is 14 cm.

25. (11) : Here, $3[x] - 5\dfrac{|x|}{x} = \begin{cases} 3[x]-5, & \text{if } x > 0 \\ 3[x]+5, & \text{if } x < 0 \end{cases}$

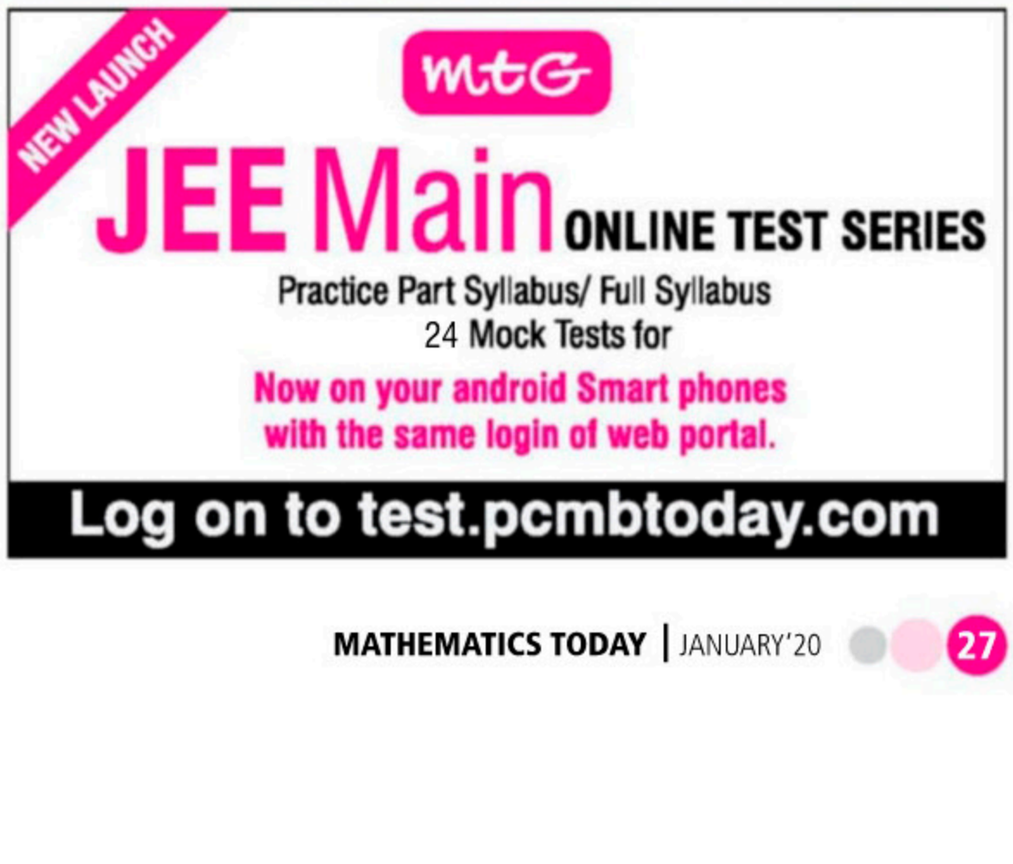
Now, $\int\limits_{-3/2}^{2} f(x)dx = \int\limits_{-3/2}^{-1} f(x)dx + \int\limits_{-1}^{0} f(x)dx + \int\limits_{0}^{1} f(x)dx + \int\limits_{1}^{2} f(x)dx$

$$= \int_{-3/2}^{-1} (3\times(-2)+5)dx + \int_{-1}^{0} (-3+5)dx + \int_{0}^{1} (0-5)dx + \int_{1}^{2} (-2)dx$$

$$= -1\left(-1 + \frac{3}{2}\right) + 2(1) + 1(-5) + (-2)1$$

$$= -\frac{1}{2} + 2 - 5 - 2 = -\frac{11}{2}$$

$\therefore \quad -2\int\limits_{-3/2}^{2} f(x)dx = 11$



5 THINGS YOU NEED TO DO BEFORE CLASS 12 PRE-BOARD EXAMS

The Central Board of Secondary Education, CBSE has released the dates for class 12 practical examinations 2020. The practical exams for class 12 are scheduled from January 1 to February 7, 2020. The pre-board exams conducted by schools are an excellent opportunity for every student to find out how effective their preparation is so far. Here are 5 things you need to do before class 12 pre-board exams 2020.

1 Pre-board exams provide real time scenario similar to the CBSE board exams. You need to pay proper attention and treat them like real board exams. It is important to be well versed with the syllabus and revise all the topics at least once before taking the pre-board exams. For this, you must focus on **NCERT textbooks and Exemplar books** as similar questions are generally asked in the boards. This will help you prepare well for the pre-boards as well as for the boards. At this time, you must practice questions from heavy-weightage topics which will help you to cover the syllabus in an efficient manner.

2 Practice from **MTG CBSE Champion Book Series**, gives you a clear view about topics which are important for CBSE Board exams. This will immensely help in your targeted revision plan and enable in scoring high.

3 Since, pre-boards are conducted just before the main board exams, so, it is essential to focus on revision and make a proper revision-schedule that covers all the subjects and topics given in the syllabus. You need to strictly follow this revision schedule and adhere to it till your board exams. Chalk out your time-table in a way that you have enough time to revise your syllabus. Also, you should set your daily targets and try to meet them. You need to be more focused and avoid delaying your daily goals. You all know that it is only through practicing which will make you proficient in solving the questions and allow you to score high in the exams. The best way to practice and revise all the topics is to solve **previous years' papers and sample papers based on latest CBSE pattern**. MTG has also released **Score More 21 sample papers**, specially designed by a very efficient team of experts and teachers for CBSE board exam 2020.

4 After covering the complete syllabus and solving numerous practice papers, you should identify your weak and strong areas. The best way to identify them is to solve as many **sample papers** as you can. You need to analyse your performance after every practice test and aim to get better in the next paper. For this, you need to solve **21 sample papers** given in **MTG's Score More Books** and can check your preparation level along with your strong and weak areas. The Self Evaluation sheet provided after each Sample Question Paper (SQP) will help you to assess your performance. The performance analysis table will help you to check where you actually stand.

5 While preparing for the exams, it is also important to know how to manage and utilize the time properly. It is seen that most of the students start writing the paper without taking care of proper time management and could not finish writing the entire paper within the given time frame. Due to this, they lose marks despite of knowing the answers correctly. Hence, it is advisable to practice solving problems in a time bound manner. The best way to learn how to manage your time during exam is to solve **CBSE sample papers** and **Previous Years' Papers**. Solving the sample papers and previous years' papers give you ample idea about how to manage time during the exam as these are designed according to the real board exam paper pattern.

We hope, the given tips will help you to prepare and perform well in your pre-boards and boards. Lastly, you should treat the pre-board exams as a part of your preparation for the CBSE board exams as the results of these exams tell you about the efficiency in your preparedness and readiness for the boards. Pre-boards are meant to help you and not stress you out and hence, make full use of this opportunity and appear for it with full preparation.

"One isn't born with perfection; it is through practice that makes him or her proficient."

All the Best!

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. If $a + b + c = 3$, then show that $a^2b^3c^2 \le \frac{3^{10}2^4}{7^7}$. Also, find the values of a, b and c when equality holds.

(Ahan Bandyopadhyay, M.P.)

Ans. For the seven numbers $\frac{a}{2}, \frac{a}{2}, \frac{b}{3}, \frac{b}{3}, \frac{b}{3}, \frac{c}{2}, \frac{c}{2}$, we have A.M. $\ge$ G.M.

$$\Rightarrow \quad \frac{2\left(\frac{a}{2}\right)+3\left(\frac{b}{3}\right)+2\left(\frac{c}{2}\right)}{7} \ge \left[\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2\right]^{\frac{1}{7}}$$

$$\Rightarrow \quad \frac{a+b+c}{7} \ge \left(\frac{a^2b^3c^2}{2^2 3^3 2^2}\right)^{\frac{1}{7}} \Rightarrow \left(\frac{3}{7}\right)^7 \ge \frac{a^2b^3c^2}{2^4 3^3}$$

[Using $a + b + c = 3$]

$$\Rightarrow \quad a^2b^3c^2 \le \left(\frac{3}{7}\right)^7 (2^4 3^3) \Rightarrow a^2b^3c^2 \le \frac{3^{10}2^4}{7^7}$$

If equality holds, then $\frac{a}{2} = \frac{b}{3} = \frac{c}{2} \Rightarrow a = c$ and $b = \frac{3}{2}a$

Since, $a + b + c = 3 \Rightarrow a + \frac{3}{2}a + a = 3 \Rightarrow a = \frac{6}{7}$

$\therefore \quad a = c = \frac{6}{7}$ and $b = \frac{9}{7}$

2. Let $f(z)$ be a polynomial in z having complex coefficients.When $f(z)$ is divided by $z^2 + z + 1$ and $z - 1$, remainders are $2\omega z + \omega$ and $5 - \omega$ respectively $\left(\text{where } \omega = \frac{-1+i\sqrt{3}}{2}\right)$. Find the remainder, when $f(z)$ is divided by $z^3 - 1$. ***(Ronit Kumar, Gujarat)***

Ans. Given,

$f(z) = \alpha(z)(z^2 + z + 1) + 2\omega z + \omega$, where $\alpha(z)$ is some polynomial.

Also $f(z) = \beta(z)(z - 1) + 5 - \omega$, where $\beta(z)$ is another polynomial.

Now, $f(\omega) = 2\omega^2 + \omega = \omega^2 - 1$ ...(i)

and $f(\omega^2) = 2\omega^3 + \omega = 2 + \omega$ ...(ii)

[$\because 1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$]

and $f(1) = 5 - \omega$ ...(iii)

Now, if $f(z)$ be divided by $z^3 - 1$, we get remainder of the form $az^2 + bz + c$, where a, b and c remains to be evaluated.

Now, $f(z) = \phi(z)(z^3 - 1) + az^2 + bz + c$

$f(\omega) = a\omega^2 + b\omega + c = \omega^2 - 1$ [From (i)]

$f(\omega^2) = a\omega + b\omega^2 + c = 2 + \omega$ [From (ii)]

$f(1) = a + b + c = 5 - \omega$ [From (iii)]

Solving these equations, we get

$$a = \frac{2\omega^2 - 2\omega + 7}{3}, b = \frac{2\omega + 5}{3}, c = \frac{\omega^2 + 6}{3}$$

Hence, required remainder $az^2 + bz + c$ is given by

$$\frac{1}{3}[(2\omega^2 - 2\omega + 7)z^2 + (2\omega + 5)z + \omega^2 + 6].$$

3. Show that the difference of the squares of the tangents to two coplanar circles from any point P in the plane of the circles varies as the perpendicular from P on their radical axis. Also, prove that the locus of a point such that the difference of the squares of the tangents from it to two given circles is constant, is a line parallel to their radical axis. ***(Harsh Mittal, Delhi)***

Ans. Let the two circles be

$S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ ...(i)

and $S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ ...(ii)

and let coordinates of P be (h, k).

$\therefore$ Radical axis of (i) and (ii) is

$2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0$ ...(iii)

Let length of tangents from $P(h, k)$ on (i) and (ii) are l_1 and l_2, then

$$l_1 = \sqrt{S_1} = \sqrt{(h^2 + k^2 + 2g_1h + 2f_1k + c_1)}$$

and $l_2 = \sqrt{S_2} = \sqrt{(h^2 + k^2 + 2g_2h + 2f_2k + c_2)}$

Consider, $l_1^2 - l_2^2 = 2(g_1 - g_2)h + 2(f_1 - f_2)k + c_1 - c_2$ ...(iv)

Let p be the perpendicular distance from $P(h, k)$ on (iii),

$$\therefore \quad p = \frac{|2(g_1 - g_2)h + 2(f_1 - f_2)k + c_1 - c_2|}{\sqrt{4(g_1 - g_2)^2 + 4(f_1 - f_2)^2}} \quad \text{...(v)}$$

From (iv) and (v), we get

$$\frac{|l_1^2 - l_2^2|}{p} = 2\sqrt{(g_1 - g_2)^2 + (f_1 - f_2)^2} = \text{constant}$$

$\therefore \quad |l_1^2 - l_2^2| \propto p$

Locus of $P(h, k)$ in (iv), is

$2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = (l_1^2 - l_2^2)$,

which is a line parallel to (iii).

MATHS MUSING

SOLUTION SET-204

1. (a) : Note that $|z_2| = 6$ represents a circle.

$|z_1 + z_2| = |z_2 - (-24 - 7i)|$ represent distance between a point on the circle $|z_2| = 6$ and the point $C(-24 - 7i)$.

$|z_1 + z_2|$ will be greatest and least at points B and A which are the end points of the diameter of the circle $|z_2| = 6$.

As $OC = 25$, $CA = OC - OA = 25 - 6 = 19$ and $CB = OC + OB = 25 + 6 = 31$.

2. (b): $I = \int_0^1 (1-x^2)^9 x^9 dx$, put $x^2 = t$

$$\therefore \quad I = \int_0^1 (1-t)^9 t^4 \frac{dt}{2} = \frac{1}{2}\frac{9!\,4!}{14!}$$

$$\therefore \quad \frac{1}{I} = \frac{2\cdot 14!}{9!\,4!} = \frac{2\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10}{24} = 20020$$

3. (c) : Let the 4 numbers p, q, r, s are in A.P. *i.e.* $p = a - 3d$, $q = a - d$, $r = a + d$, $s = a + 3d$

Now, $p + q = 4$, $r + s = 20$,

$\Rightarrow \quad p + q + r + s = 24 \Rightarrow 4a = 24 \Rightarrow a = 6$

Now, $pq = A$, $rs = B$

$p + q = 4 = 12 - 4d \Rightarrow 4d = 8 \Rightarrow d = 2$

$\therefore \quad p = 0, q = 4, r = 8, s = 12$

Now, $pq = A = (0)(4) = 0$, $rs = B = (8)(12) = 96$

$\therefore \quad (A, B) = (0, 96)$

4. (c) : Let $P(x_1, y_1)$, chord of contact is $xx_1 + yy_1 = a^2$

$$\Rightarrow \quad y = \left(-\frac{x_1}{y_1}\right)x + \frac{a^2}{y_1} \text{ touches } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$\therefore$ Locus of P is $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = \dfrac{a^2}{b^2}$

5. (d): $P(A) = 0.7$, $P(\overline{B}) = 0.6$

Now, $P(A \cap B) = P(A) - P(A \cap \overline{B}) = 0.7 - 0.5 = 0.2$

$$P(B/(A \cup \overline{B})) = \frac{0.2}{0.7 + 0.6 - 0.5} = \frac{1}{4}$$

6. (a, b, c, d) : $10 < x \le 24$

$\Rightarrow \quad 24^2 < x^2 + 100$

$\therefore \quad x^2 > 476 \quad \Rightarrow \quad x = 22, 23, 24 \qquad ...(i)$

$x > 24 > 10 \Rightarrow x^2 < 24^2 + 100$

$\therefore \quad x^2 < 676 \quad \therefore \quad x \le 25 \qquad ...(ii)$

$\therefore$ From (i) and (ii), $x = 22, 23, 24, 25$

7. (c) : ABC is an equilateral triangle of side $\sqrt{6}$.

Distance of A from BC = altitude $= \dfrac{\sqrt{3}}{2}\sqrt{6} = \dfrac{3}{\sqrt{2}}$

8. (b): The plane OBC is $\begin{vmatrix} x & y & z \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 0$

$\Rightarrow \quad 5x - y - 7z = 0$

Distance of A from face $OBC = \dfrac{18}{\sqrt{75}} = \dfrac{6\sqrt{3}}{5}$

9. (5) : $(1-x)^{10} = C_0 - C_1 x + C_2 x^2 - ... + C_{10} x^{10}$

$\therefore \quad (1-x)^{10} x^4 = C_0 x^4 - C_1 x^5 + C_2 x^6 - ... + C_{10} x^{14}$

$$S = \int_0^1 (1-x)^{10} \cdot x^4 \, dx = \frac{10!\,4!}{15!}$$

$$\therefore \quad \frac{1}{S} = \frac{15\cdot 14\cdot 13\cdot 12\cdot 11}{24} = 3\cdot 5\cdot 7\cdot 11\cdot 13 \text{ (5 primes)}$$

10. (a) : P → 3; Q → 2; R → 1; S → 4

Letters CCLLUUAS

	Words		Number of words	
(P)	2+1	$\Rightarrow$	$\binom{3}{1}\binom{4}{1}\cdot\frac{3!}{2!} = 36$	96 words
	1+1+1	$\Rightarrow$	$\binom{5}{3}\cdot 3! = 60$	
(Q)	2+2	$\Rightarrow$	$\binom{3}{2}\cdot\frac{4!}{(2!)^2} = 18$	354 words
	2+1+1	$\Rightarrow$	$\binom{3}{1}\binom{4}{2}\cdot\frac{4!}{2!} = 216$	
	1+1+1+1	$\Rightarrow$	$\binom{5}{4}\cdot 4! = 120$	
(R)	2+2+1	$\Rightarrow$	$\binom{3}{2}\cdot\binom{3}{1}\cdot\frac{5!}{2!2!} = 270$	1110 words
	2+1+1+1	$\Rightarrow$	$\binom{3}{1}\cdot\binom{4}{3}\cdot\frac{5!}{2!} = 720$	
	1+1+1+1+1	$\Rightarrow$	$5! = 120$	
(S)	2+2+2	$\Rightarrow$	$\frac{6!}{(2!)^3} = 90$	2790 words
	2+2+1+1	$\Rightarrow$	$\binom{3}{2}\cdot\binom{3}{2}\cdot\frac{6!}{(2!)^2} = 1620$	
	2+1+1+1+1	$\Rightarrow$	$\binom{3}{1}\cdot\binom{4}{4}\cdot\frac{6!}{2!} = 1080$	

FOCUS
JEE MAIN 2020
PRACTICE PAPER

with Numerical Value Type Questions

1. Tangents PA and PB are drawn to $y^2 = 4ax$ from P. If m_1 and m_2 are the slopes of the tangents satisfying $m_1^2 + m_2^2 = 4$, then the locus of P is

(a) $y^2 = 2x(2x + a)$ (b) $y^2 = 2x(2x - a)$

(c) $y^2 = x(x - a)$ (d) $y^2 = ax$

2. If four normals can be drawn to a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ from a point $(x_0, 0)$, where $x_0 > 0$, then

(a) $x_0 > ae^2$ (b) $a < x_0 < ae^2$

(c) $0 < x_0 < a$ (d) $\frac{a}{e}<x_0<ae$

3. If $0 \le \arg(z) \le \frac{\pi}{4}$, then the least value of $\sqrt{2}\,|2z-4i|$ is

(a) 6 (b) 1 (c) 4 (d) 2

4. Let $f(x)=\begin{cases}x^3-x^2+10x-5, & x\le 1\\ -2x+\log_2(b^2-2), & x>1\end{cases}$. If $f(x)$ has greatest value at $x = 1$, then $b^2 \in (2, \lambda]$, then λ is

(a) 130 (b) 103 (c) 301 (d) 310

5. The solution of the initial value problem

$(2\ln x)\frac{dy}{dx}+\frac{y}{x}=\frac{1}{y}\cos x, y>0, x>1$ and $y\left(\frac{3\pi}{2}\right)=0$

is given by $y =$

(a) $\sqrt{\frac{1-\sin x}{\ln x}}$ (b) $\sqrt{\frac{1+\sin x}{\ln x}}$

(c) $\sqrt{\frac{1-\cos x}{\ln x}}$ (d) $\sqrt{\frac{1+\cos x}{\ln x}}$

6. The number of four digit numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 that are divisible by 3, when repetition of digits is allowed, is

(a) $2^3 \times 3^2$ (b) $2^3 \times 3^3$

(c) $2^3 \times 3^4$ (d) $2^4 \times 3^3$

7. $\int\left(\frac{2a+x}{a+x}\right)\sqrt{\frac{a-x}{a+x}}dx =$

(a) $\sqrt{a^2-x^2}-2a\sqrt{\frac{a-x}{a+x}}+c$

(b) $-\sqrt{a^2-x^2}-2a\sqrt{\frac{a-x}{a+x}}+c$

(c) $\frac{1}{a}\tan^{-1}\frac{x}{a}+\ln\left|x+\sqrt{a^2-x^2}\right|+c$

(d) $\frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+\sin^{-1}\frac{x}{a}+c$

8. Let $f(x)=\int_{x-1}^{x+1}\frac{dt}{1+t^8}$. Then, the function f

(a) has no extremum

(b) has maximum value for some $x \in (0, 1)$

(c) has minimum value for some $x \in (-1, 0)$

(d) has maximum value at $x = 0$

9. The value of $\cot 7\frac{1}{2}^\circ$ is

(a) $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$ (b) $\sqrt{2}+\sqrt{3}+\sqrt{5}+\sqrt{6}$

(c) $\sqrt{3}+\sqrt{4}+\sqrt{5}+\sqrt{6}$ (d) $\sqrt{2}+\sqrt{3}+\sqrt{6}$

By : Tapas Kr. Yogi (Prime Axis Classes), Visakhapatnam Mob : 09533632105, 09949428844

10. If $\int_0^1 \alpha e^{\beta x^2} \sin(x+k)\,dx = 0$ (for some $\alpha, \beta \in R$), $\alpha \neq 0$, then the value of k can belong to

(a) $\left[\frac{\pi}{3}, \frac{5\pi}{12}\right]$ (b) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

(c) $\left[\frac{3\pi}{4}, \frac{5\pi}{6}\right]$ (d) $\left[-\frac{\pi}{2}, -\frac{\pi}{3}\right]$

11. Given $2x - y + 2z = 2$, $x - 2y + z = -4$, $x + y + \lambda z = 4$ then the value of λ such that the given system of equation has no solution, is

(a) 3 (b) 1 (c) 0 (d) –3

12. If $(x-2)y = (x-1)^{-3}$, then $\left(\frac{d^3y}{dx^3}\right)_{x=0} =$

(a) $\frac{3}{111}$ (b) $\frac{111}{3}$ (c) $\frac{8}{333}$ (d) $\frac{333}{8}$

13. The sequence $\{x_n\}$ is defined by $x_{n+1} = x_n^2 + x_n$, $x_1 = 1/2$. Then the value of $\sum_{n=1}^{2012} \frac{1}{x_n + 1}$ lies in the interval

(a) (0, 1) (b) (2012, 2013)

(c) (1, 2) (d) (2012, ∞)

14. If matrix $A = [a_{ij}]_{3\times 3}$, matrix $B = [b_{ij}]_{3\times 3}$, where $a_{ij} + a_{ji} = 0$ and $b_{ij} - b_{ji} = 0\ \forall\ i, j$, then $A^4 \cdot B^3$ is

(a) singular matrix (b) zero matrix

(c) symmetric matrix (d) skew-symmetric matrix

15. Let a point $P(h, k)$ satisfies the equation $x^3 + y^3 + 3xy = 1$. Then the locus of the point P is

(a) a straight line

(b) a circle

(c) a parabola

(d) combined equation of 3 different lines

16. If the graph of $y = ax^3 + bx^2 + cx + d$ is symmetric about the line $x = k$, then the value of $a + k$ is

(a) c (b) $c^2 - bd$

(c) $-\frac{c}{2b}$ (d) $-\frac{b}{2c}$

17. If $f(x) = 0$ is a equation whose coefficients are all ± 1 and whose roots are all real, then degree of $f(x)$ can't be equal to

(a) 1 (b) 2 (c) 3 (d) 4

18. For unit vectors $\vec{b}$ and $\vec{c}$ and any non-zero vector $\vec{a}$, the value of $\left\{\left((\vec{a}+\vec{b})\times(\vec{a}+\vec{c})\right)\times(\vec{b}\times\vec{c})\right\}\cdot(\vec{b}+\vec{c})$ is

(a) $|\vec{a}|^2$ (b) $2|\vec{a}|^2$ (c) $3|\vec{a}|^2$ (d) 0

19. Locus of the image of the point (2, 3) in the line $(x - 2y + 3) + \lambda(2x - 3y + 4) = 0$ is $(\lambda \in R)$

(a) $x^2 + y^2 - 3x - 4y - 4 = 0$

(b) $2x^2 + 3y^2 + 2x + 4y - 7 = 0$

(c) $2x^2 + 3y^2 + 2x + 4y + 7 = 0$

(d) $x^2 + y^2 - 2x - 4y + 3 = 0$

20. The statement $p \to (q \to p)$ is equivalent to

(a) $p \to (p \to q)$ (b) $p \to (p \vee q)$

(c) $p \to (p \wedge q)$ (d) $p \to (p \leftrightarrow q)$

Numerical Value Type

21. In ΔABC, $A - B = 120°$ and $R = 8r$, then $8\cos C$ is equal to_____

22. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$ $(a > 0)$ at P and Q respectively, then PQ/a =_____

23. The expression $(x + 2y)^{20}$ contain two terms with the same coefficient of the form $kx^a y^b$ and $kx^{a+1} y^{b-1}$, then the value of a is _____

24. The number of positive integral solutions of the equation $\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} = 11$ is _____

25. Let $g(x) = \frac{f(x)}{x+1}$, where $f(x)$ is differentiable on $[0, 5]$ such that $f(0) = 4$, $f(5) = -1$. There exists $c \in (0, 5)$ such that $|6\,g'(c)|$ is _____

SOLUTIONS

1. (a) : Let $P \equiv (h, k)$.

Equation of tangent to $y^2 = 4ax$ is $y = mx + \frac{a}{m}$

$\therefore \quad k = mh + \frac{a}{m} \Rightarrow m^2h + a - mk = 0$

So, $m_1 + m_2 = \frac{k}{h}$ and $m_1 \times m_2 = \frac{a}{h}$

But given that $m_1^2 + m_2^2 = 4$

$\Rightarrow \quad \frac{k^2}{h^2} - \frac{2a}{h} = 4 \Rightarrow k^2 - 2ah = 4h^2$

$\therefore$ Locus of $P(h, k)$ is $y^2 = 2ax + 4x^2 = 2x(2x + a)$

2. (a) : Equation of normal to the hyperbola is

$$\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$$

$\therefore \quad \frac{ax_0}{\sec\theta}=a^2+b^2 \Rightarrow \sec\theta=\frac{ax_0}{a^2+b^2}$

$\Rightarrow \quad \cos\theta=\frac{a^2e^2}{ax_0} \qquad (\because a^2+b^2=a^2e^2)$

So, $0<\frac{ae^2}{x_0}<1 \Rightarrow x_0>ae^2$

3. (c) : The conditions cover the region bounded by X-axis and $y = x$ least value of $|z - 2i|$ is the length of perpendicular from (0, 2) to $y = x$ which is $\sqrt{2}$.

$\therefore$ Least value of $\sqrt{2}\,|2z-4i|=4$

4. (a) : $\lim\limits_{x\to1^+} f(x)\le f(1) \Rightarrow -2+\log_2(b^2-2)\le 5$

$\Rightarrow \quad b^2 \le 130$. But $b^2 - 2 > 0 \Rightarrow b^2 > 2$

$\therefore \quad 2 < b^2 \le 130$. So, $\lambda = 130$

5. (b) : $2y\frac{dy}{dx}+y^2\left(\frac{1}{x\ln x}\right)=\frac{\cos x}{\ln x}$

$\Rightarrow \quad \frac{dz}{dx}+\frac{z}{x\ln x}=\frac{\cos x}{\ln x}$, where $z = y^2$

$\Rightarrow \quad y^2\ln x = z\ln x = \sin x + c$, where $c = 1$

$\Rightarrow \quad y=\sqrt{\frac{1+\sin x}{\ln x}} \qquad (\because \quad y > 0)$

6. (d) : The first 3 digits can be filled in 6^3 ways. After filling the first 3 digits, the last digit can be filled using 1, 2, 3, 4, 5, 6 in 6 ways. But the six numbers so formed are consecutive integers out of which only two are divisible by 3. Hence, the number of choices

$$= 2 \times 6^3 = 2^4 \times 3^3$$

7. (a) : Put $\theta=\cos^{-1}\frac{x}{a}$ $(-a<x<a)$. Then $0 < \theta < \pi$

and $I=-a\int\frac{(2+\cos\theta)(1-\cos\theta)}{1+\cos\theta}d\theta$

$$=-a\int\left\{(1-\cos\theta)+\frac{1-\cos\theta}{1+\cos\theta}\right\}d\theta$$

$$=-a\left(2\tan\frac{\theta}{2}-\sin\theta\right)+c=\sqrt{a^2-x^2}-2a\sqrt{\frac{a-x}{a+x}}+c$$

8. (d) : $f(x)=\int_0^{x+1}\frac{dx}{1+x^8}-\int_0^{x-1}\frac{dx}{1+x^8}$

$\Rightarrow \quad f'(x)=\frac{1}{1+(1+x)^8}-\frac{1}{1+(1-x)^8}$

$$=\frac{(1-x)^8-(1+x)^8}{\{1+(1+x)^8\}\{1+(1-x)^8\}}$$

$\therefore \quad f'(x) = 0 \Leftrightarrow x = 0$. Also $x < 0 \Rightarrow f'(x) > 0$ and $x > 0 \Rightarrow f'(x) < 0$

$\therefore \quad f$ has maximum value at $x = 0$.

9. (a) : $\cot 7\frac{1}{2}^\circ=\frac{\cos 7\frac{1}{2}^\circ}{\sin 7\frac{1}{2}^\circ}=\frac{2\cos^2 7\frac{1}{2}^\circ}{\sin 15^\circ}=\frac{1+\cos 15^\circ}{\sin 15^\circ}$

Putting $\sin 15^\circ=\frac{\sqrt{3}-1}{2\sqrt{2}}$ and $\cos 15^\circ=\frac{\sqrt{3}+1}{2\sqrt{2}}$, we get

$\cot 7\frac{1}{2}^\circ=\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

10. (c) : Value of definite integral can be zero when either the limits coincide or f becomes identically zero for given interval or $f(x) = 0$ has atleast one root in the given interval. Here, $\sin(x + k) = 0$ in $x \in [0, 1]$.

11. (b) : Since given system has no solution $\therefore \Delta = 0$ and any one amongst $\Delta_x, \Delta_y, \Delta_z$ is non-zero.

Let $\begin{vmatrix}2&-1&2\\1&-2&1\\1&1&\lambda\end{vmatrix}=0$ and $\Delta_z=\begin{vmatrix}2&-1&2\\1&-2&-4\\1&1&4\end{vmatrix}=6\neq 0$

12. (d) : Given, $y=\frac{1}{(x-1)^3(x-2)}$

$$=-\frac{1}{(x-1)^3}-\frac{1}{(x-1)^2}-\frac{1}{x-1}+\frac{1}{x-2}$$

Resolving into partial fractions

$= -(x - 1)^{-3} - (x - 1)^{-2} - (x - 1)^{-1} + (x - 2)^{-1}$

Now, differentiate 3 times.

13. (c) : $\frac{1}{x_{n+1}}=\frac{1}{x_n(x_n+1)}=\frac{1}{x_n}-\frac{1}{x_n+1}$

$\therefore \sum\limits_{n=1}^{2012}\frac{1}{x_n+1}=\frac{1}{x_1}-\frac{1}{x_{2013}}=2-\frac{1}{x_{2013}}=1+\left(1-\frac{1}{x_{2013}}\right)$

$= 1 + f$ where $0 < f < 1$

14. (a) : Here, $a_{ij} + a_{ji} = 0 \Rightarrow A^T = -A$

and $b_{ij} - b_{ji} = 0 \Rightarrow B^T = B$ and A, B are 3×3 matrices.

Hence, $|A| = 0 \Rightarrow |A^4B^3| = 0$

$\Rightarrow \quad A^4 \cdot B^3$ is singular matrix.

15. (a) : $x^3 + y^3 + 3xy = 1 \Rightarrow x^3 + y^3 + (-1)^3 = 3xy(-1)$

$\therefore \quad x = y = -1$ or $x + y - 1 = 0$

Hence, locus is an isolated point (or) a straight line.

16. (c) : The graph of a cubic cannot be symmetric about $x = k$

$\Rightarrow$ $a = 0$ and $bx^2 + cx + d$ is symmetric about

$x = \frac{-c}{2b} \Rightarrow k = \frac{-c}{2b}$; $a + k = -\frac{c}{2b}$

17. (d) : Let the equation be

$a_0x^n + a_1x^{n-1} + \ldots + a_n = 0 \Rightarrow \sum\alpha_i = \pm 1$

$\sum\alpha_i\alpha_j = \pm 1 \Rightarrow \sum\alpha_i^2 = 1 + 2 = 3 \Rightarrow n \le 3$

18. (d) : Given, $\{(\vec{a}+\vec{b})\times(\vec{a}+\vec{c}))\times(\vec{b}\times\vec{c})\}\cdot(\vec{b}+\vec{c})$

$= \left\{((\vec{b}\times\vec{c})\cdot(\vec{a}+\vec{b}))(\vec{a}+\vec{c}) - ((\vec{b}\times\vec{c})\cdot(\vec{a}+\vec{c}))\vec{a}+\vec{b}\right\}\cdot(\vec{b}+\vec{c})$

$= \left[\vec{b} \quad \vec{c} \quad \vec{a}\right]\left((\vec{c}-\vec{b})\cdot(\vec{c}+\vec{b})\right)$

$= \left[\vec{b} \quad \vec{c} \quad \vec{a}\right]\left(|\vec{c}|^2 - |\vec{b}|^2\right) = 0$

19. (d) : The family $(x - 2y + 3) + \lambda(2x - 3y + 4) = 0$ of lines are concurrent at point $P(1, 2)$.

Let $B(h, k)$ be the image of $A(2, 3)$ w.r.t. the above variable line, then $AP = BP$.

20. (b)

21. (7) : $R = 8 \times 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$\Rightarrow \quad 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{1}{16}$

$\Rightarrow \quad \left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right]\sin\frac{C}{2} = \frac{1}{16}$

$\Rightarrow \quad \sin\frac{C}{2} = \frac{1}{4} \Rightarrow \cos C = \frac{7}{8}$

22. (3) : $\frac{x}{1} = \frac{y-(-a)}{1} = \frac{z}{1}$ and $\frac{x-(-a)}{1} = \frac{y}{1/2} = \frac{z}{1/2}$

Let coordinates of P and Q are respectively

$(t, t-a, t), \left(s-a, \frac{s}{2}, \frac{s}{2}\right)$.

$\therefore \quad$ Dr.'s of $PQ = \left(s-t-a, \frac{s}{2}-t+a, \frac{s}{2}-t\right)$

So, $\frac{s-t-a}{2} = \frac{\frac{s}{2}-t+a}{1} = \frac{\frac{s}{2}-t}{2} \Rightarrow s = 2a$ and $t = 3a$

$\therefore \quad PQ = 3a$

23. (6) : Let $T_{r+1} = {}^{20}C_r \cdot x^{20-r}(2y)^r = kx^a y^b$

$$T_{s+1} = {}^{20}C_s \cdot x^{20-s}(2y)^s = kx^{a+1}y^{b-1}$$

For given condition, $r - s = 1$

and ${}^{20}C_{1+s} \cdot 2^{1+s} = {}^{20}C_s \cdot 2^s \Rightarrow s = 13$

$\therefore \quad r = 14 \Rightarrow a = 6$

24. (3) : After expanding we get, $x^3 + y^3 + z^3 = 10$

$\because \quad x, y, z \in N \Rightarrow (2, 1, 1), (1, 2, 1), (1, 1, 2)$ are possibilities.

25. (5) : $g'(c) = \frac{\frac{f(5)}{6} - \frac{f(0)}{1}}{5}$ (From L.M.V.T.)

$= \frac{-\frac{1}{6} - 4}{5} = \frac{-25}{6\times 5} = -\frac{5}{6}$

JEE Main to go multilingual, preference to languages of states with majority candidates : MHRD

The Ministry of Human Resource Development (MHRD) is planning to conduct the Joint Entrance Examination (JEE Main) in multiple languages and will give preference to states with the maximum number of applicants. "The MHRD will give preference to the regional languages of states with a greater number of applications in the last five years," the secretary office communicated.

HRD secretary R Subrahmanyam stated that there can be technical limitations in conducting JEE Main in multiple languages. "Since JEE Main is held in a computer-based mode, there are some limitations which need to be addressed, first. We are committed to conducting JEE Main in multiple languages and the process will take time," Subrahmanyam commented. The National Testing Agency (NTA) will have to develop software in regional languages, the HRD office informed.

Going by the maximum number of candidates, Marathi and Telugu would be added first in the JEE Main. From Maharashtra, 1.1 lakh applications were received from April 2019 exam. while from Andhra Pradesh and Telangana, the combined number of candidates stood at 1.60 lakh.

The NTA is set to conduct the JEE 2020 in English and Hindi. However, for the centres in Gujarat, Daman and Diu, Dadra and Nagar Haveli, the languages would be English, Hindi and Gujarati. Explaining the inclusion of Gujarati, NTA said the translation of JEE Main question paper in this language continued on the state's request. None of the other states has approached the agency to provide the question paper in their regional language. Earlier, the Central Board of Secondary Education (CBSE) conducted the JEE Main in Marathi, Gujarati and Urdu languages till 2014, but withdrew Urdu and Marathi in 2016.

OLYMPIAD CORNER

1. Consider the infinite sum

$$S = \frac{a_0}{10^0} + \frac{a_1}{10^2} + \frac{a_2}{10^4} + \frac{a_3}{10^6} + \ldots,$$

where the sequence $\{a_n\}$ is defined by $a_0 = a_1 = 1$, and the recurrence relation $a_n = 20a_{n-1} + 12a_{n-2}$ for all positive integers $n \geq 2$. If $\sqrt{S}$ can be expressed in the form $\frac{a}{\sqrt{b}}$, where a and b are relatively prime positive integers, determine the ordered pair (a, b).

2. Let X be a finite set and $E(X)$ be the collection of subsets of X with an even number of elements. A real-valued function f is defined on $E(X)$ such that $f(D) > 1990$ for at least one D in $E(X)$ and $f(A \cup B) = f(A) + f(B) - 1990$ for all disjoint A and B in $E(X)$. Prove that X may be partitioned into two disjoint subsets P and Q such that $f(S) > 1990$ for all non-empty S in $E(P)$ and $f(T) \leq 1990$ for all T in $E(Q)$.

3. A circle with centre O and radius R is drawn on a paper, and A is a given point in the circle with $OA = a$. Fold the paper to make a point A' on the circumference coincident with point A, then a crease line is left on the paper. Find out the set of all points on such crease lines, when A' goes through every point on the circumference.

4. A sequence is formed by the following rules : $s_1 = a$, $s_2 = b$ and $s_{n+2} = s_{n+1} + (-1)^n s_n$ for all $n \geq 1$. If $a = 3$ and b is an integer less than 1000, what is the largest value of b for which 2015 is a member of the sequence? Justify your answer.

5. Ten players participate in a ping-pong tournament, in which every two players play against each other exactly once. If player i beats player j, player j beats player k and player k beats player i, then the set $\{i, j, k\}$ is called a triangle. Let w_i and l_i denote the number of games won and lost, respectively, by the i^{th} player. Suppose that, whenever i beats j, then $l_i + w_j \geq 8$. Prove that there are exactly 40 triangles in the tournament.

6. Find all real numbers x such that

$$x = \left(x - \frac{1}{x}\right)^{1/2} + \left(1 - \frac{1}{x}\right)^{1/2}$$

7. $ABCD$ is a convex quadrilateral and O is the intersection of its diagonals. Let L, M, N be the mid-points of DB, BC, CA respectively. Suppose that AL, OM, DN are concurrent. Show that either $AD \parallel BC$ or $[ABCD] = 2[OBC]$.

SOLUTIONS

1. We have

$$S - \frac{20S}{10^2} - \frac{12S}{10^4} = \left(\frac{a_0}{10^0} + \frac{a_1}{10^2} + \frac{a_2}{10^4} + \frac{a_3}{10^6} + \ldots.\right) - \left(\frac{20a_0}{10^2} + \frac{20a_1}{10^4} + \frac{20a_2}{10^6} + \frac{20a_3}{10^8} + \ldots.\right) - \left(\frac{12a_0}{10^4} + \frac{12a_1}{10^6} + \frac{12a_2}{10^8} + \frac{12a_3}{10^{10}} + \ldots.\right)$$

$$= \frac{a_0}{10^0} + \frac{a_1}{10^2} - \frac{20a_0}{10^2} + \frac{a_2 - 20a_1 - 12a_0}{10^4} + \frac{a_3 - 20a_2 - 12a_1}{10^6} + \frac{a_4 - 20a_3 - 12a_2}{10^8} + \ldots.$$

Since, $a_n - 20a_{n-1} - 12a_{n-2} = 0$ for all positive integers $n \geq 2$, we have

$$S - \frac{20S}{10^2} - \frac{12S}{10^4} = \frac{a_0}{10^0} + \frac{a_1}{10^2} - \frac{20a_0}{10^2}, \quad \ldots(i)$$

and substituting $a_0 = a_1 = 1$ in (i), we have

$$\frac{7988S}{10000} = \frac{81}{100} \Rightarrow S = \frac{2025}{1997}$$

Hence, $\sqrt{S} = \frac{45}{\sqrt{1997}}$

So, the desired ordered pair is $(a, b) = (45, 1997)$.

2. Among the subsets in $E(X)$, choose P for which $f(P)$ is maximum. If there are more than one such choice, take one with the smallest number of elements. If there are several such subsets,

take an arbitrary one. Define $Q = X - P$. Since $f(D) > 1990$ for some D in $E(X)$, $f(P) > 1990$. Let $S \neq P$ be a non-empty subset in $E(P)$. Then $P - S$ also belongs to $E(P)$ and has a smaller number of elements than P. Hence $f(P - S) < f(P)$.

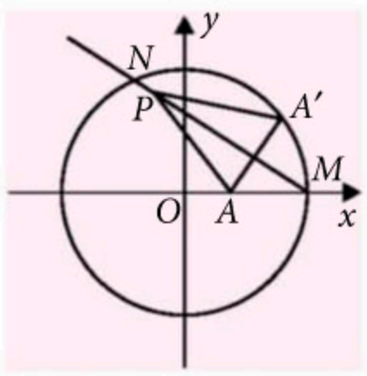
From $f(P) = f(S \cup (P - S)) < f(S) + f(P - S) - 1990$, $f(S) > 1990$.

Now, let T be any subset in $E(Q)$. Then $f(P \cup T) \leq f(P)$.

From $f(P \cup T) = f(P) + f(T) - 1990$,

$f(T) \leq 1990$.

3. Establish an xy-coordinate system as in the diagram with $A(a, 0)$ given. Then, the crease line MN is the perpendicular bisector of segment AA' when $A'(R\cos a, R\sin a)$ is made coincident with A by folding the paper.



Let $P(x, y)$ be any point on MN, then $|PA'| = |PA|$. *i.e.*, $(x - R\cos a)^2 + (y - R\sin a)^2 = (x - a)^2 + y^2$.

Then, $$\frac{x\cos a + y\sin a}{\sqrt{x^2 + y^2}} = \frac{R^2 - a^2 + 2ax}{2R\sqrt{x^2 + y^2}}.$$

$$\Rightarrow \quad \sin(\theta + a) = \frac{R^2 - a^2 + 2ax}{2R\sqrt{x^2 + y^2}},$$

where $\sin\theta = \dfrac{x}{\sqrt{x^2 + y^2}}, \cos\theta = \dfrac{y}{\sqrt{x^2 + y^2}}$.

So, $$\left|\frac{R^2 - a^2 + 2ax}{2R\sqrt{x^2 + y^2}}\right| \leq 1$$

Squaring both sides, we get, $$\frac{(2x - a)^2}{R^2} + \frac{4y^2}{R^2 - a^2} \geq 1$$

So, the set we want consists of all the points on the border or outside the ellipse is

$$\frac{(2x - a)^2}{R^2} + \frac{4y^2}{R^2 - a^2} = 1$$

4. Working out the first few terms gives us an idea of how the given sequence develops:

n	s_{2n-1}	s_{2n}
1	a	b
2	$b - a$	$2b - a$
3	b	$3b - a$
4	$2b - a$	$5b - 2a$
5	$3b - a$	$8b - 3a$
6	$5b - 2a$	$13b - 5a$
7	$8b - 3a$	$21b - 8a$

It appears that the coefficients in the even terms form a Fibonacci sequence and from the 5^{th} term, every odd term is a repeat of the third term before it.

These observations are true for the entire sequence since, for $m \geq 1$, we have:

$s_{2m+2} = s_{2m+1} + s_{2m}$

$s_{2m+3} = s_{2m+2} - s_{2m+1} = s_{2m}$

$s_{2m+4} = s_{2m+3} + s_{2m+2} = s_{2m+2} + s_{2m}$

So, defining $F_1 = 1$, $F_2 = 2$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$, we have $s_{2n} = bF_n - aF_{n-2}$ for $n \geq 3$. Since $a = 3$ and $b < 1000$, none of the first five terms of the given sequence equal 2015. So we are looking for integral solutions of $bF_n - 3F_{n-2} = 2015$ for $n \geq 3$.

$s_6 = 3b - 3 = 2015$, has no solution.

$s_8 = 5b - 6 = 2015$, has no solution.

$s_{10} = 8b - 9 = 2015$ implies $b = 253$.

For $n \geq 6$ we have $b = 2015/F_n + 3F_{n-2}/F_n$. Since F_n increases, we have $F_n \geq 13$ and $F_{n-2}/F_n < 1$ for $n \geq 6$.

Hence $b < 2015/13 + 3 = 158$.

So the largest value of b is 253.

5. Let P_i denote the i^{th} player, $i = 1, 2, ..., 10$. Without loss of generality, assume that $w_1 \leq w_2 \leq ... \leq w_{10}$. Let $M = \max_{1 \leq i \leq 10} w_i = w_{10}$ and $m = \min_{1 \leq i \leq 10} w_i = w_1$. Since $w_i + l_i = 9$ for all i, the condition $l_i + w_j \geq 8$, whenever P_i beats P_j, is equivalent to

$$w_i \leq w_j + 1 \qquad ...(i)$$

Clearly

$$\sum_{i=1}^{10} w_i = \binom{10}{2} = 45 \qquad ...(ii)$$

We first show that

$(w_1, w_2, \ldots, w_{10}) = (4, 4, 4, 4, 4, 5, 5, 5, 5, 5)$

Since P_1 beats m players and loses to $9 - m$ others, we get from (i) and (ii),

$45 \leq m + Mm + (9 - m)(m + 1)$

or $\quad 36 \leq (M - m + 9)m \qquad ...(iii)$

Similarly, since P_{10} beats M players and loses to $9 - M$ others, we have

$45 \geq M + M(M - 1) + (9 - M)m = M^2 + (9 - M)m \qquad ...(iv)$

Since $M \leq 9$, from (iii) we obtain $36 \leq (18 - m)m$, which implies $m \geq 3$. From (iv) we get $M^2 \leq 45$, *i.e.* $M \leq 6$. If $M = 6$, then from (iv), $3m \leq 9$ or $m \leq 3$ and so $m = 3$. Similarly, if $m = 3$ then from (iii) $M \geq 6$ and so $M = 6$. Since 10×45, $M \neq m$. Therefore, the only possibilities are $(M, m) = (6, 3)$ and $(5, 4)$.

Thus $(M, m) = (5, 4)$ and $(4, 4, 4, 4, 4, 5, 5, 5, 5, 5)$ is the only possible solution. By exhibiting the

tournament graphically, one can see that this is indeed a solution.

Now, there is a theorem in graph theory which states that the number of transitive triples [3 players P_i, P_j and P_k such that P_i beats P_j, P_j beats P_k and P_i beats P_k] in a tournament with n vertices and score sequence $(w_1, w_2, ..., w_n)$ is $\sum_{i=1}^{n} w_i(w_i - 1)/2$. It follows that, with $n = 10$, the number of triangles (cyclic triples) is

$$\binom{10}{3} - \frac{1}{2}\sum_{i=1}^{10} w_i(w_i - 1)$$

$$= 120 - \frac{1}{2}(5)(5)(5-1) - \frac{1}{2}(5)(4)(4-1) = 40.$$

This completes the proof.

6. Since $\left(x - \frac{1}{x}\right)^{1/2} \geq 0$ and $\left(1 - \frac{1}{x}\right)^{1/2} \geq 0$, then

$$0 \leq \left(x - \frac{1}{x}\right)^{1/2} + \left(1 - \frac{1}{x}\right)^{1/2} = x$$

Note that $x \neq 0$. Else, $\frac{1}{x}$ would not be defined, so $x > 0$.

Squaring both sides gives,

$$x^2 = \left(x - \frac{1}{x}\right) + \left(1 - \frac{1}{x}\right) + 2\sqrt{\left(x - \frac{1}{x}\right)\left(1 - \frac{1}{x}\right)}$$

$$\Rightarrow \quad x^2 = x + 1 - \frac{2}{x} + 2\sqrt{x - 1 - \frac{1}{x} + \frac{1}{x^2}}$$

Multiplying both sides by x and rearranging, we get

$$x^3 - x^2 - x + 2 = 2\sqrt{x^3 - x^2 - x + 1}$$

$$\Rightarrow \quad (x^3 - x^2 - x + 1) - 2\sqrt{x^3 - x^2 - x + 1} + 1 = 0$$

$$\Rightarrow \quad (\sqrt{x^3 - x^2 - x + 1} - 1)^2 = 0 \Rightarrow \sqrt{x^3 - x^2 - x + 1} = 1$$

$$\Rightarrow \quad x^3 - x^2 - x + 1 = 1 \Rightarrow x(x^2 - x - 1) = 0$$

$$\Rightarrow \quad x^2 - x - 1 = 0 \qquad [\text{Since } x \neq 0]$$

Thus, $x = \frac{1 \pm \sqrt{5}}{2}$. We must check to see if these are indeed solutions.

Let $\alpha = \frac{1+\sqrt{5}}{2}$, $\beta = \frac{1-\sqrt{5}}{2}$. Note that $\alpha + \beta = 1$, $\alpha\beta = -1$ and $\alpha > 0 > \beta$.

Since $\beta < 0$, β is not a solution.

Now, if $x = \alpha$, then

$$\left(\alpha - \frac{1}{\alpha}\right)^{1/2} + \left(1 - \frac{1}{\alpha}\right)^{1/2}$$

$= (\alpha + \beta)^{1/2} + (1 + \beta)^{1/2}$ (Since $\alpha\beta = -1$)

$= 1^{1/2} + (\beta^2)^{1/2}$ (Since $\alpha + \beta = 1$ and $\beta^2 = \beta + 1$)

$= 1 - \beta$ (Since $\beta < 0$)

$= \alpha$ (Since $\alpha + \beta = 1$)

So $x = \alpha$ is the unique solution to the equation.

7. Let O be the origin of a coordinate system where A, B, C, D are represented by $(a, 0)$, $(0, b)$, $(c, 0)$, $(0, d)$ with a, b positive and c, d negative. Thus the point L is $\left(0, \frac{(b+d)}{2}\right)$, M is $\left(\frac{c}{2}, \frac{b}{2}\right)$, N is $\left(\frac{(a+c)}{2}, 0\right)$ and

$AL : (b + d)x + 2ay - a(b + d) = 0$

$OM : bx - cy = 0$

$DN : 2dx + (a + c)y - d(a + c) = 0$

These lines are concurrent if and only if

$$\begin{vmatrix} b & -c & 0 \\ b+d & 2a & -a(b+d) \\ 2d & a+c & -d(a+c) \end{vmatrix} = 0.$$

This equation reduces (after some manipulation) to $(ab - cd)[(a - c)(b - d) + 2bc] = 0$.

Consequently, either

(a) $ab = cd$, in which case $AD \parallel BC$, or

(b) $\frac{1}{2}(a - c)(b - d)\sin\alpha = 2\left(-\frac{1}{2}bc\sin\alpha\right)$

(where $\alpha = \angle AOB$), in which case $[ABCD] = 2[OBC]$.

SAMURAI SUDOKU

ANSWER · DECEMBER 2019

3	8	6	2	9	4	1	5	7				8	5	2	6	3	9	7	1	4
2	4	5	7	1	8	9	6	3				4	1	3	5	2	7	8	6	9
9	1	7	5	3	6	4	2	8				7	6	9	4	8	1	3	5	2
4	2	3	1	8	5	6	7	9				2	8	1	7	9	3	6	4	5
1	5	8	9	6	7	3	4	2				3	7	6	2	5	4	9	8	1
6	7	9	3	4	2	8	1	5				9	4	5	1	6	8	2	7	3
5	6	4	8	2	3	7	9	1	5	4	3	6	2	8	3	1	5	4	9	7
7	3	1	4	5	9	2	8	6	1	9	7	5	3	4	9	7	6	1	2	8
8	9	2	6	7	1	5	3	4	8	2	6	1	9	7	8	4	2	5	3	6
						8	1	9	7	3	2	4	5	6						
						6	7	2	4	5	9	8	1	3						
						3	4	5	6	8	1	9	7	2						
7	8	2	4	6	9	1	5	3	2	6	4	7	8	9	4	3	2	1	5	6
9	5	6	3	1	7	4	2	8	9	7	5	3	6	1	7	9	5	8	2	4
3	4	1	2	8	5	9	6	7	3	1	8	2	4	5	8	1	6	7	9	3
5	2	3	9	7	8	6	1	4				1	2	3	6	4	9	5	8	7
4	1	7	6	5	3	2	8	9				8	5	4	2	7	3	6	1	9
6	9	8	1	4	2	7	3	5				6	9	7	1	5	8	3	4	2
1	7	4	5	3	6	8	9	2				4	1	2	3	8	7	9	6	5
2	6	5	8	9	4	3	7	1				9	3	8	5	6	4	2	7	1
8	3	9	7	2	1	5	4	6				5	7	6	9	2	1	4	3	8

Exam on 2nd Feb 2020

WBJEE MOCK TEST PAPER 2020

CATEGORY-I (Q. 1 to Q. 50)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ¼ mark will be deducted.

1. If $(1 + i)(1 + 2i)(1 + 3i) \ldots (1 + ni) = a + ib$, then $2 \cdot 5 \cdot 10 \ldots (1 + n^2)$ is equal to

(a) $a^2 + b^2$ (b) $\sqrt{a^2 + b^2}$

(c) $\sqrt{a^2 - b^2}$ (d) $a^2 - b^2$

2. If $1+\sin x+\sin^2 x+\ldots\infty = 4+2\sqrt{3}, 0 < x < \pi$, then x equals to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ or $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$

3. Let $f : R \to R$ such that $f(x) = x^3 + 5x + 1$, then

(a) f is neither one-one nor onto

(b) f is one-one but not onto

(c) f is onto but not one-one

(d) f is one-one and onto

4. The value of $\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{5\pi}{8}+\sin^2\frac{7\pi}{8}$ is equal to

(a) 2 (b) 1/8 (c) 1/4 (d) 1

5. The value of ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to

(a) ${}^{47}C_5$ (b) ${}^{52}C_5$ (c) ${}^{52}C_4$ (d) ${}^{52}C_3$

6. If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from the set A to set B is

(a) 2^9 (b) 9^2 (c) 3^2 (d) $2^9 - 1$

7. In a triangle ABC, $(a + b + c)(b + c - a) = \lambda bc$, if

(a) $\lambda < 0$ (b) $0 \le \lambda \le 4$

(c) $0 \le \lambda < 4$ (d) $0 < \lambda \le 4$

8. The circles $x^2 + y^2 + 6x + 6y = 0$ and $x^2 + y^2 - 12x - 12y = 0$

(a) cut orthogonally

(b) touch each other internally

(c) intersect at two points

(d) touch each other externally

9. If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then $x \in$

(a) $(1, 2)$ (b) $(2, \infty)$ (c) $(-2, 0)$ (d) $(1, \infty)$

10. If the roots of the quadratic equation $mx^2 - nx + k = 0$ are $\tan 33°$ and $\tan 12°$, then the value of $\frac{2m+n+k}{m} =$

(a) 0 (b) 1 (c) 2 (d) 3

11. If $\cos^{-1}\sqrt{x^2+2x+1} + \text{cosec}^{-1}\sqrt{x^2+2x+1} = \frac{\pi}{2}$, $x \neq 0$, then value of $\sec^{-1}\left(\frac{x}{2}\right) - \sin^{-1}\left(\frac{x}{2}\right)$ is equal to

(a) $-\frac{3\pi}{2}$ (b) $-\frac{\pi}{2}$ (c) $3\pi/2$ (d) $\pi/2$

12. Let $P(n) = 5^n - 2^n$, $P(n)$ is divisible by 3λ where λ and n both are odd positive integers, then the least value of n and λ will be

(a) 13 (b) 11 (c) 1 (d) 5

13. Let $f(x) = \log_e x + \log_x e$, then the domain of the function $\frac{1}{\sqrt{|f(x)| - f(x)}}$ is

(a) $(0, 1)$ (b) $(1, \infty)$ (c) $(1, e)$ (d) $(-\infty, \infty)$

14. Let $f:(4,6) \to (6,8)$ be a function defined by $f(x) = x + \left[\frac{x}{2}\right]$ (where $[\cdot]$ denotes the greatest integer function), then $f^{-1}(x)$ is equal to

(a) $x - \left[\frac{x}{2}\right]$ (b) $-x - 2$

(c) $x - 2$ (d) $\frac{1}{x + \left[\frac{\pi}{6}\right]}$

15. Locus of a point which divides chord at a distance of 1 unit from the centre of the circle $x^2 + y^2 = 10$ in the ratio 2 : 1 is

(a) $x^2 + y^2 = 2$ (b) $x^2 + y^2 = 4$

(c) $x^2 + y^2 = 8$ (d) $x^2 + y^2 = 16$

16. $\lim\limits_{x\to 3}\dfrac{1-\cos(x^2-x-6)}{(x-3)^2}=$

(a) $-\dfrac{1}{2}$ (b) $\dfrac{25}{2}$ (c) $\dfrac{1}{2}$ (d) $-\dfrac{25}{2}$

17. If the orthocentre and centroid of a triangle are $(-3, 5)$ and $(3, 3)$, then the circumcentre is

(a) $(6, 2)$ (b) $(0, 8)$ (c) $(6, -2)$ (d) $(0, 4)$

18. Let a, b, c are the sides of a triangle and $(\sin A + \sin B + \sin C)(\sin A + \sin C - \sin B) = \mu \sin A \sin C$, then the range of μ is

(a) $[0, 1]$ (b) $[-4, 4]$ (c) $(0, 4)$ (d) $[0, 4]$

19. If $f(x)=\begin{cases}\dfrac{x-[x]}{x-3}, & x<4\\ b, & x=4\\ \dfrac{a|x^2-5x+6|}{(x-3)}, & x>4\end{cases}$

is continuous at $x = 4$, then

(a) $a = 0, b = 1$ (b) $a = 1/2, b = 1$

(c) $a = -1/2, b = 1$

(d) $f(x)$ is continuous everywhere for any real a and b

20. The principal argument of the complex number

$$z=\frac{1+\sin\frac{\pi}{3}+i\cos\frac{\pi}{3}}{1+\sin\frac{\pi}{3}-i\cos\frac{\pi}{3}}$$ is

(a) $\pi/3$ (b) $\pi/6$ (c) $2\pi/3$ (d) $\pi/2$

21. $A(3, 2, 0)$, $B(5, 3, 2)$ and $C(-9, 6, -3)$ are the vertices of a triangle ABC. If the bisector of $\angle BAC$ meets BC at D, then coordinates of D are

(a) $\left(\dfrac{19}{8},\dfrac{57}{16},\dfrac{17}{16}\right)$ (b) $\left(-\dfrac{19}{8},\dfrac{57}{16},\dfrac{17}{16}\right)$

(c) $\left(\dfrac{19}{8},-\dfrac{57}{16},\dfrac{17}{16}\right)$ (d) none of these

22. Area of the triangle with vertices (a, b), (x_1, y_1) and (x_2, y_2) where a, x_1, x_2 are in G.P. with common ratio r and b, y_1, y_2 are in G.P. with common ratio s, is

(a) $\dfrac{ab}{2}(r-1)(s-1)(s-r)$

(b) $\dfrac{1}{2}ab(r+1)(s+1)(s-r)$

(c) $\dfrac{1}{2}ab(r+1)(s+1)(r-s)$

(d) $\dfrac{1}{2}ab(r-1)(s+1)(r-s)$

23. If $f(x)=\dfrac{|x+2|}{\tan^{-1}(x+2)}$, then which is not true?

(a) $\lim\limits_{x\to -2^+} f(x)=\lim\limits_{x\to -2^-} f(x)$

(b) $\lim\limits_{x\to -2^+} f(x)=1$

(c) f is continuous except at $x = -2$

(d) f is non differentiable at $x = -2$

24. The derivative of the function

$$f(x)=\cos^{-1}\left\{\frac{1}{\sqrt{13}}(2\cos x-3\sin x)\right\}+\sin^{-1}\left\{\frac{1}{\sqrt{13}}(2\sin x+3\cos x)\right\}$$

with respect to $\sqrt{1+x^2}$ is

(a) $2x$ (b) $2\sqrt{1+x^2}$

(c) $\dfrac{2}{x}\sqrt{1+x^2}$ (d) $\dfrac{2x}{\sqrt{1+x^2}}$

25. The distance between the planes $\vec{r}\cdot(2\hat{i}-3\hat{j}+6\hat{k})=5$ and $\vec{r}\cdot(6\hat{i}-9\hat{j}+18\hat{k})+20=0$ is

(a) 2/3 (b) 4/3 (c) 5/3 (d) 7/3

26. If $\theta=\dfrac{\pi}{4n}$, where n is positive integer, then the value of $\tan\theta\tan 2\theta\tan 3\theta.....\tan(2n-1)\theta=$

(a) 1 (b) $\tan\theta$ (c) $\cot\theta$ (d) 0

27. The area bounded by the curve $x^2 + 2x + y - 3 = 0$, the x-axis and the tangent at the point where it meets the y-axis is

(a) $\dfrac{7}{12}$ sq. units (b) $\dfrac{12}{7}$ sq. units

(c) $\dfrac{7}{6}$ sq. units (d) $\dfrac{6}{7}$ sq. units

28. If $(x^2 + y^2)dy = xydx$, $y(1) = 1$ and $y(x_0) = e$, then $x_0 =$

(a) $\sqrt{2(e^2-1)}$ (b) $\sqrt{2(e^2+1)}$

(c) $\sqrt{3}\cdot e$ (d) $\sqrt{\dfrac{e^2+1}{2}}$

29. The probability distribution of a random variable is

$$P(X)=\begin{cases}\dfrac{X}{15}; X=1,2,3,4,5\\ 0, \text{ otherwise}\end{cases}.$$

Then $P\left\{\dfrac{1}{2}<X<\dfrac{5}{2}\,|\,X>1\right\}=$

(a) 1/5 (b) 1/7 (c) 2/15 (d) 1/15

30. The slope of the straight line which is both tangent and normal to the curve $4x^3 = 27y^2$ is

(a) ± 1 (b) $\pm\frac{1}{2}$ (c) $\pm\frac{1}{\sqrt{2}}$ (d) $\pm\sqrt{2}$

31. The integral $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x^2}}$ represents the function

(a) $6\left[\sqrt[3]{x^2}-\sqrt[3]{x}+\log\left|1+\sqrt[3]{x}\right|\right]+c$

(b) $3\sqrt[3]{x}-6\sqrt[6]{x}+6\log\left|1+\sqrt[6]{x}\right|+c$

(c) $3\sqrt[3]{x}+6\sqrt[6]{x}+6\log\left|1+\sqrt[6]{x}\right|+c$

(d) $6\sqrt[3]{x^2}-3\sqrt[3]{x}+6\log\left|1+\sqrt[3]{x}\right|+c$

32. The slopes of three sides of a triangle are –1, –2, 3. If the orthocentre of the triangle is the origin, then locus of the centroid of the triangle is

(a) $2x + 9y = 0$ (b) $2x - 9y = 0$
(c) $9x - 2y = 0$ (d) $9x + 2y = 0$

33. The number of integral terms in the expansion of $(2\sqrt{5}+\sqrt[6]{7})^{642}$ is

(a) 105 (b) 107 (c) 321 (d) 108

34. If the slope of a chord OP of the parabola $y^2 = 4ax$ (where O is the vertex) is $\tan\alpha$, then the length of the chord is

(a) $4a\tan\alpha\sec\alpha$ (b) $4a\tan\alpha\cos\alpha$
(c) $4a\cot\alpha\operatorname{cosec}\alpha$ (d) $4a\sin\alpha\tan\alpha$

35. A five digit number is formed by writing the digits 1, 2, 3, 4, 5 in a random order without repetition. Then the probability that the number is divisible by 4 is

(a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) $\frac{5}{6}$ (d) $\frac{5}{18}$

36. The standard deviation of 50 values of a variable x is 15; if each value of the variable is divided by (–3); then the standard deviation of the new set of 50 values of x will be

(a) 15 (b) –5 (c) 5 (d) –15

37. For three vectors $\vec{a},\vec{b},\vec{c}$

$\left[\vec{a}\times(\vec{b}\times\vec{c})\,\vec{b}\times(\vec{c}\times\vec{a})\,\vec{c}\times(\vec{a}\times\vec{b})\right]=$

(a) 2 (b) 4 (c) 8 (d) 0

38. If $0 \le [x] < 2, -1 \le [y] < 1$ and $1 \le [z] < 3$, ($[\cdot]$ denotes the greatest integer function), then the maximum value of determinant $\Delta = \begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is

(a) 2 (b) 6 (c) 4 (d) 3

39. The equation of the plane through the line of intersection of $\vec{r}\cdot(\hat{i}-2\hat{j}+2\hat{k})=1$ and $\vec{r}\cdot(2\hat{i}-\hat{j})+2=0$ and perpendicular to $\vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k})+9=0$ is

(a) $\vec{r}\cdot(-5\hat{i}+\hat{j}+2\hat{k})=7$ (b) $\vec{r}\cdot(5\hat{i}+\hat{j}+2\hat{k})=7$
(c) $\vec{r}\cdot(5\hat{i}-\hat{j}-2\hat{k})=7$ (d) none of these

40. $\int\frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}dx$ is equal to

(a) $\sin x-6\tan^{-1}(\sin x)+c$
(b) $\sin x-\frac{2}{\sin x}-6\tan^{-1}(\sin x)+c$
(c) $\sin x-\frac{2}{\sin x}+5\tan^{-1}(\sin x)+c$
(d) $\sin x+\frac{2}{\sin x}+6\tan^{-1}(\sin x)+c$

41. If for a variable line $\frac{x}{a}+\frac{y}{b}=1$, the condition $a^{-2}+b^{-2}=c^{-2}$ (c is a constant) is satisfied, then the locus of foot of the perpendicular drawn from the origin to this line is

(a) $x^2+y^2=\frac{c^2}{2}$ (b) $x^2+y^2=2c^2$
(c) $x^2+y^2=c^2$ (d) $x^2-y^2=c^2$

42. For a differentiable function f,

$\lim\limits_{h\to 0}\frac{\{f(x+h)\}^2-\{f(x)\}^2}{2h}=$

(a) $\{f'(x)\}^2$ (b) $\frac{1}{2}\{f'(x)\}^2$
(c) $\frac{1}{2}[\{f'(x)\}^2-\{f(x)\}^2]$ (d) $f(x)f'(x)$

43. If a random variable X follows binomial distribution with mean 2 and $E(X^2)= 28/5$, then $(n, p) =$

(a) $\left(10,\frac{1}{5}\right)$ (b) $\left(5,\frac{1}{10}\right)$ (c) $\left(10,\frac{4}{5}\right)$ (d) $\left(5,\frac{4}{5}\right)$

44. The area (in sq. units) between the curves $y = xe^x$ and $y = xe^{-x}$ and the line $x = 1$ is

(a) $2e$ (b) e (c) $2/e$ (d) $1/e$

45. The domain of the function $f(x)=\frac{\sin^{-1}(3-x)}{\log(|x|-2)}$ is

(a) $[2, 4]$ (b) $(2, 3)\cup(3, 4]$
(c) $[2, \infty)$ (d) $(-\infty, -3)\cup[2, \infty)$

46. If $\frac{1}{ab+ac},\frac{1}{bc+ba},\frac{1}{ca+cb}$ are in H.P., then a, b, c are in

(a) A.P. (b) G.P.
(c) H.P. (d) none of these

47. A cone is inscribed in a sphere of radius r. The height of the cone when its volume is maximum is

(a) $2r$ (b) $\frac{\sqrt{3}r}{2}$ (c) $\frac{2r}{\sqrt{3}}$ (d) $\frac{4r}{3}$

48. An ellipse has eccentricity 1/2 and one focus at $S\left(\frac{1}{2}, 1\right)$. Its one directrix is the common tangent, (nearer to S) to the circle $x^2 + y^2 = 1$ and $x^2 - y^2 = 1$. The equation of the ellipse in standard form is

(a) $9\left(x-\frac{1}{3}\right)^2+12(y-1)^2=1$

(b) $12\left(x-\frac{1}{3}\right)^2+9(y-1)^2=1$

(c) $\frac{\left(x-\frac{1}{2}\right)^2}{12}+\frac{(y-1)^2}{9}=1$

(d) $3\left(x+\frac{1}{2}\right)^2+4(y-1)^2=1$

49. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that each is equally inclined to the other at an angle θ, then $\theta \in$

(a) $\left[0, \frac{\pi}{2}\right]$ (b) $\left[\frac{\pi}{2}, \pi\right]$

(c) $\left[0, \frac{2\pi}{3}\right]$ (d) $[0, \pi]$

50. If a variate assumes the values 0, 1, 2, ..., n with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_n$ then mean square deviation about the value $x = 0$ is

(a) $\frac{n(n-1)}{2}$ (b) $\frac{n^2(n-1)}{4}$

(c) $\frac{n(n+1)}{4}$ (d) $\frac{n(n+1)}{2}$

CATEGORY-II (Q. 51 to Q. 65)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ½ mark will be deducted.

51. The number of ways in which the letters of the word 'COMBINE' can be arranged so that the word begin and end with a vowel, is

(a) 30 (b) 504 (c) 360 (d) 720

52. Let $f: R \to R$ defined by $f(x) = x^3 + x^2 + 100x + 5\sin x$, then f is

(a) many-one onto (b) many-one into

(c) one-one onto (d) one-one into

53. If $x>0, y>0, z>0$ such that $\log_2 x+\log_4 y+\log_4 z=2$, $\log_9 x+\log_3 y+\log_9 z=2, \log_{16} x+\log_{16} y+\log_4 z=2$, then $x + y + z =$

(a) 12 (b) $\frac{353}{24}$ (c) $\frac{355}{24}$ (d) 15

54. The middle term in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$ is

(a) $\frac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}{n!}\cdot x^n$ (b) $\frac{1\cdot3\cdot5\cdot\ldots\cdot(2n+1)}{n!}\cdot 2^n$

(c) $\frac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}{(n+1)!}\cdot 2^n$ (d) $\frac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}{n!}\cdot 2^n$

55. A line passing through the point of intersection of $x + y = 4$ and $x - y = 2$ makes an angle $\tan^{-1}\frac{3}{4}$ with the x-axis. It intersects the parabola $y^2 = 4(x - 3)$ at points (x_1, y_1) and (x_2, y_2) respectively. Then $|x_1 - x_2| =$

(a) $\frac{16}{9}$ (b) $\frac{32}{9}$ (c) $\frac{40}{9}$ (d) $\frac{80}{9}$

56. The sum to n-terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\cdots$

(a) $2^n - n - 1$ (b) $1 - 2^{-n}$

(c) $n + 2^{-n} - 1$ (d) $2^n - 1$

57. If C is the centre and A, B are two points on the conic $4x^2 + 9y^2 - 8x - 36y + 4 = 0$ such that $\angle ACB = \frac{\pi}{2}$, then $(CA)^{-2} + (CB)^{-2}$ is equal to

(a) $\frac{13}{36}$ (b) $\frac{36}{13}$ (c) $\frac{16}{33}$ (d) $\frac{33}{16}$

58. Two candidates A and B appeared for an interview for two vacancies. The probabilities of their selection are $\frac{1}{4}$ and $\frac{1}{3}$ respectively. The probability that one of them will be selected is

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{7}{12}$ (d) $\frac{5}{12}$

59. If $t=\frac{2\sqrt{2}-(1+\sqrt{3})}{\sqrt{3}-1}$ and $f(x)=\frac{2x}{1-x^2}$, $g(x)=\frac{3x-x^3}{1-3x^2}$, then $\frac{d}{dt}(f(g(t)))$ is

(a) 1 (b) 2 (c) 0 (d) –1

60. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$ and $f(1) = 1$, $f(p + q) = f(p) \cdot f(q)\ \forall\ p, q \in R$, then $x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{x+y+z}{x^{f(1)} + y^{f(2)} + z^{f(3)}} =$

(a) 0 (b) 1 (c) 2 (d) 3

61. If $f(x)=\int_{x^2}^{x^3}\frac{dt}{\log t}, x>0$, then

(a) $f(x)$ is maximum at $x=1$

(b) $f(x)$ is an increasing function $\forall\ x\in R^+$ only

(c) $f(x)$ is minimum at $x=1$

(d) $f(x)$ is neither maximum nor minimum at $x=1$

62. If $y-\cos x\frac{dy}{dx}=y^2(1-\sin x)\cos x,\ y(0)=1,$ then $y\left(\frac{\pi}{3}\right)=$

(a) 1 (b) 2 (c) 1/2 (d) $\sqrt{3}$

63. If $z+\frac{1}{z}=2\cos 15^\circ$, then value of $z^{2017}+\frac{1}{z^{2017}}$ equals

(a) $2\cos 15^\circ$ (b) $2\sin 15^\circ$

(c) $\sin 15^\circ$ (d) none of these

64. Let PQ be a double ordinate of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If O be the centre of the hyperbola and OPQ is an equilateral triangle, then the eccentricity e is

(a) $>\sqrt{3}$ (b) >2 (c) $>\frac{2}{\sqrt{3}}$ (d) $>\frac{1}{\sqrt{3}}$

65. Let $\frac{d}{dx}(f(x))=\frac{e^{\sin x}}{x}, x>0$. If $\int_1^4\frac{2e^{\sin x^2}}{x}dx$ $=f(k)-f(1)$, then one of the possible value of k is

(a) -4 (b) 0 (c) 2 (d) 16

CATEGORY-III (Q. 66 to Q. 75)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong, but there is no negative marking for the same and zero mark will be awarded.

66. In a ΔABC, $\tan A$ and $\tan B$ are the roots of the equation $ab(x^2+1)=c^2x$, where a, b and c are the sides of the triangle. Then

(a) $\tan(A-B)=\frac{a^2-b^2}{2ab}$ (b) $\cot C=0$

(c) $\sin^2A+\sin^2B=1$ (d) none of these

67. Let $f: R\to R$ such that $f(x)=[x]^2+[x+1]-3$, where $[x]$ = the greatest integer $\le x$. Then

(a) $f(x)$ is a many-one and into function

(b) $f(x)=0$ for infinite number of values of x

(c) $f(x)=0$ for only two real values of x

(d) none of these

68. If the arithmetic mean of two positive numbers a and b $(a>b)$ is twice their G.M., then $a:b=$

(a) $6+\sqrt{7}:6-\sqrt{7}$ (b) $2+\sqrt{3}:2-\sqrt{3}$

(c) $5+\sqrt{6}:5-\sqrt{6}$ (d) none of these

69. Let $n\in N$ then $x(x^{n-1}-na^{n-1})+a^n(n-1)$ is divisible by $(x-a)^2$ when

(a) $n=2$ (b) $n=3$

(c) $n=1$ (d) none of these

70. If $\tan^2x+2\tan x\tan 2y=\tan^2y+2\tan y\tan 2x=k$, then which of the following is correct?

(a) $k=1$ (b) $\tan x=\tan y$

(c) $\tan x=-\tan y$ (d) 0

71. Let A and B be two events such that $P(A\cap B)=\frac{1}{3}$, $P(A\cup B)=\frac{5}{6}$ and $P(\bar{A})=\frac{1}{2}$. Then

(a) A, B are independent

(b) A, B are mutually exclusive

(c) $P(A)=P(B)$

(d) $P(B)\le P(A)$

72. If $A=\begin{bmatrix}1&0\\2&1\end{bmatrix}$ and $A^{2012}=\begin{bmatrix}a&b\\c&d\end{bmatrix}$, then which of the following is/are correct?

(a) $a=d$ (b) $a+b+c+d=4026$

(c) $a^2+b^2+d^2=2$ (d) $b=2012$

73. If all the three roots of $az^3+bz^2+cz+d=0$ have negative real parts $(a, b, c\in R)$, then

(a) $ab>0$ (b) $bc>0$

(c) $ad>0$ (d) $bc+ad<0$

74. Let $a_n=\underbrace{111\ldots1}_{n\text{ times}}$, then

(a) a_{912} is not prime (b) a_{951} is not prime

(c) a_{480} is not prime (d) a_{91} is not prime

75. If $f(x)=2+|\sin^{-1}x|$, it is

(a) continuous no where

(b) continuous everywhere in its domain

(c) differentiable no where in its domain

(d) not differentiable at $x=0$

SOLUTIONS

1. (a): Given that, $(1+i)(1+2i)(1+3i)\ldots(1+ni)=a+ib$

Now, $|(1+i)(1+2i)(1+3i)\ldots(1+ni)|=|a+ib|$

$\Rightarrow |1+i|^2|1+2i|^2|1+3i|^2\ldots|1+ni|^2=|a+ib|^2$

$\Rightarrow (1+1)(1+4)(1+9)\ldots(1+n^2)=a^2+b^2$

$\Rightarrow 2\times5\times10\times\ldots\times(1+n^2)=a^2+b^2$

2. (d) : $1+\sin x+\sin^2 x+...\infty = 4+2\sqrt{3}, 0<x<\pi$

$\Rightarrow \frac{1}{1-\sin x} = 4+2\sqrt{3} \Rightarrow 1-\sin x = \frac{1}{4+2\sqrt{3}} = \frac{2-\sqrt{3}}{2}$

$\Rightarrow \sin x = 1-\frac{2-\sqrt{3}}{2} = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

3. (d) **4. (a)** **5. (c)** **6. (a)**

7. (b) : We have $(a + b + c)(b + c - a) = \lambda bc$

$\Rightarrow (b + c)^2 - a^2 = \lambda bc \Rightarrow b^2 + c^2 - a^2 = (\lambda - 2)bc$

$\Rightarrow \frac{b^2+c^2-a^2}{2bc} = \frac{\lambda-2}{2} \Rightarrow \cos A = \frac{\lambda-2}{2}$

$\because -1 \le \cos A \le 1 \Rightarrow -1 \le \frac{\lambda-2}{2} \le 1 \Rightarrow 0 \le \lambda \le 4$

8. (d) : For the circle, $x^2 + y^2 + 6x + 6y = 0$, the centre is at $(-3, -3)$ and radius = $3\sqrt{2}$

Also the centre and radius of the circle $x^2 + y^2 - 12x - 12y = 0$ is $(6, 6)$ and $6\sqrt{2}$ respectively.

Distance between the centres of the circles

$= \sqrt{81+81} = 9\sqrt{2}$ = sum of the radii

$\therefore$ The two circles touch each other externally.

9. (b) : $\log_{0.3}(x - 1) = \log_{0.09}(x - 1)^2$

$\therefore \log_{0.09}(x - 1)^2 < \log_{0.09}(x - 1)$

$\Rightarrow (x - 1)^2 > x - 1$ [$\because 0.09 < 1$]

$\Rightarrow (x - 1)(x - 2) > 0 \Rightarrow x > 2$ [$\because x - 1 > 0$]

10. (d) : Given that $\tan 33°$ and $\tan 12°$ are the roots of the equation $mx^2 - nx + k = 0$

$\therefore \tan 33° + \tan 12° = \frac{n}{m}$ and $\tan 33° \tan 12° = \frac{k}{m}$

Now, $\frac{2m+n+k}{m} = 2 + \frac{n}{m} + \frac{k}{m}$

$= 2 + (\tan 33° + \tan 12°) + (\tan 33° \times \tan 12°)$

$= 2 + 1 = 3$ [$\because \tan(33° + 12°) = \tan 45° = 1$]

11. (c): $\because \cos^{-1}\sqrt{x^2+2x+1} + \operatorname{cosec}^{-1}\sqrt{x^2+2x+1} = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}\sqrt{x^2+2x+1} + \sin^{-1}\left(\frac{1}{\sqrt{x^2+2x+1}}\right) = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}|x+1| - \cos^{-1}\frac{1}{|x+1|} = 0 \ (x \ne -1)$

$\Rightarrow |x+1| = \frac{1}{|x+1|} \Rightarrow |x+1|^2 = 1$

or, $x+1 = \pm 1 \Rightarrow x = -2, 0 \Rightarrow x = -2$ [$\because x \ne 0$]

Then, $\sec^{-1}\left(\frac{x}{2}\right) - \sin^{-1}\left(\frac{x}{2}\right) = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

12. (c): Given, $P(n) = 5^n - 2^n$

Let $n = 1$, so, $P(1) = 3 = 3 \times 1 = 3\lambda$, in this case, $\lambda = 1$

Similarly, if $n = 5$, then $P(5) = 5^5 - 2^5$

$\Rightarrow P(5) = 3125 - 32 = 3093 = 3 \times 1031$, in this case, $\lambda = 1031$

Similarly, we can check the result for other cases and find that the least value of λ and n is 1.

13. (a): The function $\frac{1}{\sqrt{|f(x)| - f(x)}}$ will be defined when

$|f(x)| - f(x) > 0$

$\Rightarrow |f(x)| > f(x) \Rightarrow f(x) < 0$

$\Rightarrow \log_e x + \log_x e < 0$ ($\because f(x) = \log_e x + \log_x e$)

$\Rightarrow \log_e x + \frac{1}{\log_e x} < 0 \Rightarrow \frac{(\log_e x)^2 + 1}{\log_e x} < 0$

$\Rightarrow \log_e x < 0 \Rightarrow 0 < x < 1$

14. (c): Here, we have $4 < x < 6 \Rightarrow 2 < \frac{x}{2} < 3 \ \therefore \left[\frac{x}{2}\right] = 2$

So, $f(x) = x + \left[\frac{x}{2}\right] = x + 2, x \in (4,6)$

Let $y \in (6, 8)$ such that $f(x) = y$. Now, $f(x) = y$ gives

$y = x + 2 \Rightarrow x = y - 2 \Rightarrow f^{-1}(y) = y - 2 \Rightarrow f^{-1}(x) = x - 2$

15. (a): Let $P(h, k)$ divides the chord AB in the ratio 2 : 1. $OD \perp AB$, where O is the centre of the circle and D is the foot of the perpendicular drawn from the centre O.

Now, $OB = \sqrt{10}, OD = 1$, So, $DB = \sqrt{10-1} = 3$ and $AB = 6$

$\because \overline{AP} : \overline{PB} = 2:1 \ \therefore PB = \frac{1}{3}AB = 2 \ \therefore DP = DB - PB = 3 - 2 = 1$

Now, from ΔODP, we have

$OP^2 = OD^2 + DP^2 \Rightarrow h^2 + k^2 = 1 + 1 = 2$

$\therefore$ Locus of the point (h, k) is $x^2 + y^2 = 2$

16. (b) **17. (a)**

18. (c): $\because (\sin A + \sin B + \sin C)(\sin A + \sin C - \sin B)$

$= \mu \sin A \sin C$

$\Rightarrow (a + b + c)(a + c - b) = \mu ac$

$\Rightarrow (a + c)^2 - b^2 = \mu ac \Rightarrow a^2 + c^2 - b^2 + 2ac = \mu ac$

$\Rightarrow \frac{a^2+c^2-b^2}{2ac} + 1 = \frac{\mu}{2} \Rightarrow 1 + \cos B = \frac{\mu}{2}$

$\Rightarrow \cos^2\frac{B}{2} = \frac{\mu}{4} \Rightarrow 0 < \frac{\mu}{4} < 1$ ($\because \angle B \ne 0°$ and $\angle B \ne 180°$)

$\therefore 0 < \mu < 4$

19. (b) **20. (b)**

21. (a): D divides BC in the ratio $AB : AC$ *i.e.* 3 : 13

Therefore coordinates of D are

$\left(\frac{3\times(-9)+13\times5}{3+13}, \frac{3\times6+13\times3}{3+13}, \frac{3\times(-3)+13\times2}{3+13}\right) = \left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$

22. (a) **23. (a)** **24. (c)**

25. (c): The given equation of planes are

$\vec{r} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) = 5$ and $\vec{r} \cdot (6\hat{i} - 9\hat{j} + 18\hat{k}) + 20 = 0$

Now changing to cartesian form of the above plane we get,

$2x - 3y + 6z - 5 = 0$ and $6x - 9y + 18z + 20 = 0$

Now distance of the plane $6x - 9y + 18z + 20 = 0$ from a point (x_1, y_1, z_1) which lies on the plane $2x - 3y + 6z - 5 = 0$ is

$\frac{|6x_1 - 9y_1 + 18z_1 + 20|}{\sqrt{6^2 + (-9)^2 + 18^2}}$

$= \frac{|3(2x_1 - 3y_1 + 6z_1 - 5) + 15 + 20|}{21} = \frac{0 + 15 + 20}{21} = \frac{35}{21} = \frac{5}{3}$ units

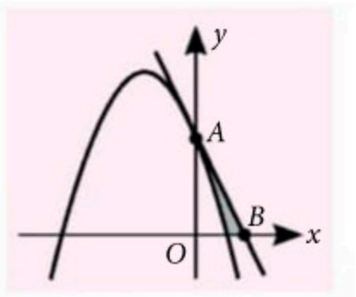
26. (a): $\tan\theta \tan 2\theta \tan 3\theta ..... \tan(2n-1)\theta$

$\left[\because \theta = \frac{\pi}{4n} \ \therefore 2n\theta = \frac{\pi}{2}$ and $\tan(2n-1)\theta = \tan(2n\theta - \theta) = \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta\right]$

$= \tan\theta \tan((2n - 1)\theta) \tan 2\theta \tan((2n - 2)\theta) .... \tan n\theta \cot n\theta$

$= (\tan\theta\cot\theta)(\tan 2\theta\cot 2\theta) .... (\tan n\theta\cot n\theta) = 1 \cdot 1 \cdot 1 .... \cdot 1 = 1$

27. (a): The equation of the curve is $y-4=-(x+1)^2$. The curve meets y-axis at $(0, 3)$ and x-axis at $(-3, 0)$ and $(1, 0)$. The tangent at $(0, 3)$ is $y-3=-2x$.



The required area

$$= \text{Area of } \Delta OAB - \int_0^1 \{4-(x+1)^2\}dx$$

$$= \frac{9}{4} - \left\{4x - \frac{(x+1)^3}{3}\right\}_0^1 = \frac{9}{4} - \left\{\left(4-\frac{8}{3}\right)+\frac{1}{3}\right\} = \frac{7}{12} \text{ sq. units}$$

28. (c)

29. (b) : $P(X)=\begin{cases}\frac{X}{15}, X=1,2,3,4,5\\ 0, \text{ otherwise}\end{cases}$

$$\therefore\ P(X=1 \text{ or } 2)=P(X=1)+P(X=2)=\frac{1}{15}+\frac{2}{15}=\frac{1}{5}$$

$$\text{Now, } P\left(\frac{1}{2}<X<\frac{5}{2}\,|\,X>1\right)=\frac{P\left\{\left(\frac{1}{2}<X<\frac{5}{2}\right)\cap X>1\right\}}{P(X>1)}$$

$$=\frac{P\{(X=1 \text{ or } 2)\cap(X>1)\}}{P(X>1)} = \frac{P(X=2)}{1-P(X=1)} = \frac{\frac{2}{15}}{1-\frac{1}{15}} = \frac{1}{7}$$

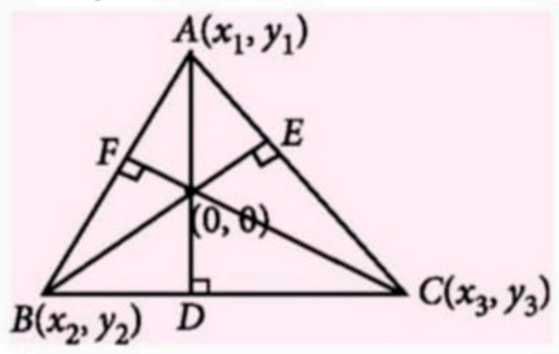
30. (d)

31. (b) : Let $I=\int \frac{dx}{\sqrt{x}+\sqrt[3]{x^2}}$ [Put $x = t^6$]

$$=\int \frac{6t^5dt}{t^3+t^4} = 6\int \frac{t^2dt}{t+1} = 6\int\left(t-1+\frac{1}{t+1}\right)dt$$

$$= 3t^2 - 6t + 6\log|t+1| + c = 3\sqrt[3]{x} - 6\sqrt[6]{x} + 6\log|\sqrt[6]{x}+1| + c$$

32. (b) : Let ABC be the triangle and slope of $BC = -1$, $AC = -2$ and $AB = 3$.



$\therefore$ Slope of $AD = 1$

$\Rightarrow\ y_1 = x_1$

Slope of BE

$$= \frac{1}{2} \Rightarrow y_2 = \frac{1}{2}x_2$$

$$\Rightarrow\ x_2 = 2y_2$$

Slope of $CF = -\frac{1}{3} \Rightarrow y_3 = -\frac{1}{3}x_3 \Rightarrow x_3 = -3y_3$

Also, $y_2 - y_3 = -1(x_2 - x_3)$;

$y_3 - y_1 = -2(x_3 - x_1)$; $y_1 - y_2 = 3(x_1 - x_2)$

Solving for x_2, x_3, y_1, y_2, y_3 in terms of x_1, we get

$$x_2 = \frac{4x_1}{5}, x_3 = \frac{9x_1}{5}, y_1 = x_1, y_2 = \frac{2x_1}{5}, y_3 = -\frac{3x_1}{5}$$

Now, centroid $= (h, k) \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$$\Rightarrow\ 3h = x_1 + \frac{4x_1}{5} + \frac{9x_1}{5} \Rightarrow 3h = \frac{18x_1}{5} \quad ...(i)$$

$$\text{and } 3k = x_1 + \frac{2x_1}{5} - \frac{3x_1}{5} \Rightarrow 3k = \frac{4x_1}{5} \quad ...(ii)$$

From (i) & (ii), we get $2h = 9k$

$\therefore$ Locus of (h, k) is $2x = 9y$.

33. (d) : The general term is

$$t_{r+1} = {}^{642}C_r(2\sqrt{5})^{642-r}(\sqrt[6]{7})^r = {}^{642}C_r\, 2^{642-r}5^{321-\frac{r}{2}}.7^{\frac{r}{6}}$$

The above term will be integral if and only if r is a multiple of 6.

Thus, r takes the values 0, 6, 12, 18, ..., 642.

Therefore the number of integral terms in the given expansion is 108.

34. (c)

35. (b) : Let A be the event that the 5-digits numbers formed by the digits 1,2,3,4,5 is divisible by 4.

Here, the number of exhaustive cases is 5! = 120.

Number is divisible by 4 when the last two digits of it are 12 or 24 or 32 or 52.

Thus in a 5-digit number the remaining 3 digits can arrange among themselves in 3! ways.

Therefore number of favourable cases = 3! × 4 = 6 × 4 = 24

$$\therefore\ P(A) = \frac{24}{120} = \frac{1}{5}$$

36. (c): Let $y_i = -\frac{1}{3}x_i$ where $i = 1, 2, ..., 50$ and $\sigma_x = 15$

$$\therefore\ \bar{y} = -\frac{1}{3}\bar{x}$$

$$\text{Now, } y_i - \bar{y} = -\frac{1}{3}(x_i - \bar{x}) \Rightarrow (y_i - \bar{y})^2 = \frac{1}{9}(x_i - \bar{x})^2$$

$$\Rightarrow \frac{1}{50}\sum_{i=1}^{50}(y_i - \bar{y})^2 = \frac{1}{9}\cdot\frac{1}{50}\sum_{i=1}^{50}(x_i - \bar{x})^2$$

$$\Rightarrow \sigma_y^2 = \frac{1}{9}\sigma_x^2 \Rightarrow \sigma_y = \frac{1}{3}\sigma_x = \frac{1}{3}\times 15 = 5$$

37. (d)

38. (c): $\Delta = \begin{vmatrix}[x]+1 & [y] & [z]\\ [x] & [y]+1 & [z]\\ [x] & [y] & [z]+1\end{vmatrix} = \begin{vmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ [x] & [y] & [z]+1\end{vmatrix}$

$\{R_1 \to R_1 - R_2, R_2 \to R_2 - R_3\}$

$= 1([z] + 1 + [y]) + 1([x]) = [x] + [y] + [z] + 1$

$\therefore$ Maximum value is 1 + 0 + 2 + 1 = 4.

39. (a): The equation of plane through the line of intersection of $\vec{r}\cdot(\hat{i}-2\hat{j}+2\hat{k})=1$ and $\vec{r}\cdot(2\hat{i}-\hat{j})+2=0$ is

$$[\vec{r}\cdot(\hat{i}-2\hat{j}+2\hat{k})-1]+\lambda[\vec{r}\cdot(2\hat{i}-\hat{j})+2]=0$$

$$\Rightarrow\ \vec{r}\cdot[(1+2\lambda)\hat{i}+(-2-\lambda)\hat{j}+2\hat{k}]=1-2\lambda \quad ...(i)$$

Since (i) is perpendicular to $\vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k})+9=0$

$\therefore$ $1(1 + 2\lambda) + (-2 - \lambda)1 + 2 \cdot 2 = 0 \Rightarrow \lambda = -3$

Required equation of plane is $\vec{r}\cdot(-5i+j+2k)=7$

40. (b)

41. (c): $\frac{x}{a}+\frac{y}{b}=1 \Rightarrow bx+ay=ab$ ...(i)

Now, if (h, k) be the foot of the perpendicular drawn from the origin on (i), then $h=\frac{ab^2}{a^2+b^2}, k=\frac{a^2b}{a^2+b^2}$.

$$\therefore \quad h^2+k^2=\frac{a^2b^4}{(a^2+b^2)^2}+\frac{a^4b^2}{(a^2+b^2)^2}=\frac{a^2b^2}{a^2+b^2}=c^2$$

$$\left(\because \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}\right)$$

$\therefore$ Locus of the point (h, k) is $x^2 + y^2 = c^2$.

42. (d)

43. (a): Let a random variable X follows a binomial distribution with parameter n and p.

Given that mean $= E(X) = np = 2$ and $E(X^2) = \frac{28}{5}$.

$$\therefore \quad npq = E(X^2)-[E(X)]^2 = \frac{28}{5}-2^2 = \frac{8}{5}.$$

Now, $\frac{npq}{np} = \frac{8}{5} \div 2 = \frac{4}{5} \Rightarrow q = \frac{4}{5}$

$$\therefore \quad p=\frac{1}{5} \text{ and hence } n=10$$

44. (c)

45. (b) : $f(x) = \dfrac{\sin^{-1}(3-x)}{\log(|x|-2)}$

Let $g(x) = \sin^{-1}(3 - x) \Rightarrow -1 \le 3 - x \le 1$

Domain of $g(x)$ is $[2, 4]$

and let $h(x) = \log(|x| - 2) \Rightarrow |x| - 2 > 0$

$\Rightarrow \quad |x| > 2 \Rightarrow x < -2$ or $x > 2 \Rightarrow (-\infty, -2) \cup (2, \infty)$

We know that

$$(g/h)(x) = \frac{g(x)}{h(x)} \forall x \in D_1 \cap D_2 - \{x \in R : h(x) = 0\}$$

$\therefore$ Domain of $f(x) = (2, 4] - \{3\} = (2, 3) \cup (3, 4]$.

46. (c) **47. (d)**

48. (a): For the circle $x^2 + y^2 = 1$ and rectangular hyperbola $x^2 - y^2 = 1$, one common tangent is evidently $x = 1$, the other being $x = -1$.

We require in standard form the ellipse with focus at $S\left(\frac{1}{2}, 1\right)$ and directrix $x = 1$ which is

$$\left(x-\frac{1}{2}\right)^2+(y-1)^2=\left(\frac{1}{2}\right)^2(1-x)^2$$

$$\Rightarrow \quad \frac{3x^2}{4}-\frac{x}{2}+(y-1)^2=0 \quad \Rightarrow \frac{3}{4}\left(x-\frac{1}{3}\right)^2+(y-1)^2=\frac{1}{12}$$

$$\Rightarrow \quad 9\left(x-\frac{1}{3}\right)^2+12(y-1)^2=1$$

49. (c)

50. (c): Mean of the distribution is

$$\bar{x}=\frac{0\cdot {}^nC_0+1\cdot {}^nC_1+2\cdot {}^nC_2+\ldots+n\cdot {}^nC_n}{{}^nC_0+{}^nC_1+\ldots+{}^nC_n}=\frac{\sum_{r=0}^{n} r.{}^nC_r}{2^n}=\frac{n\cdot 2^{n-1}}{2^n}=\frac{n}{2}$$

Again, $\frac{1}{N}\sum_{i=1}^{n} f_i x_i^2 = \frac{1}{2^n}\sum_{r=0}^{n} r^2\, {}^nC_r$ [where $N = \sum f$]

$$=\frac{n(n+1)2^{n-2}}{2^n}=\frac{n(n+1)}{4}$$

51. (d) : The word 'COMBINE' is consisting of 7 letters. There are 3 vowels namely- E, I and O.

The two vowels for beginning and ending place can be arranged in ${}^3P_2$ ways. For each of the ${}^3P_2$ ways the remaining 5 letters can be arranged in 5! ways.

Therefore required number is ${}^3P_2 \times 5! = 720$.

52. (c): $f(x) = x^3 + x^2 + 100x + 5 \sin x$

$\therefore \quad f'(x) = 3x^2 + 2x + 100 + 5 \cos x$

$=3x^2 + 2x + 94 + (6 + 5\cos x) > 0$

$f(x)$ is an increasing function and consequently a one-one function.

Clearly $f(-\infty) = -\infty$, $f(\infty) = \infty$ and $f(x)$ is continuous, therefore range $f = R$ = codomain.

Hence, f is onto.

53. (b) **54. (d)**

55. (b) : Given that the line is passing through the point (3, 1) (the point of intersection of the straight lines $x + y = 4$ and $x - y = 2$) and its slope is 3/4.

$\therefore$ The equation of the line is

$$y-1=\frac{3}{4}(x-3) \quad \Rightarrow \quad 3x-4y=5$$

Now, putting $y=\frac{3x-5}{4}$ in the equation of the parabola, we get

$$\left(\frac{3x-5}{4}\right)^2=4(x-3) \quad \Rightarrow \quad 9x^2-94x+217=0$$

Let x_1 and x_2 are the roots of the equation.

$$\therefore \quad x_1+x_2=\frac{94}{9} \text{ and } x_1x_2=\frac{217}{9}$$

$$\therefore \quad (x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2 = \frac{1024}{81} \quad \Rightarrow \quad |x_1-x_2|=\frac{32}{9}$$

56. (c)

57. (a): The equation can be written as $4(x - 1)^2 + 9(y - 2)^2 = 36$ which is an ellipse centred at (1, 2).

If CA makes an angle θ with the major axis, then

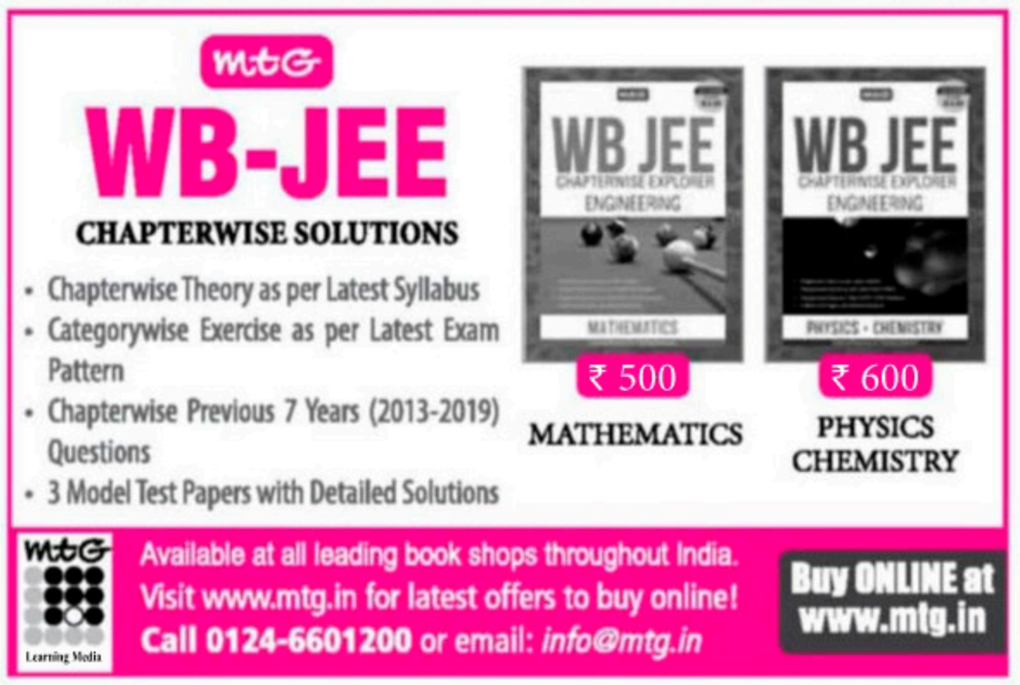
$$A \equiv [1+CA\cos\theta, 2+CA\sin\theta],$$

$$B \equiv \left[1+CB\cos\left(\frac{\pi}{2}+\theta\right), 2+CB\sin\left(\frac{\pi}{2}+\theta\right)\right]$$

As A and B are two points on the conic, so

$CA^2(4\cos^2\theta + 9\sin^2\theta) = 36$ and $CB^2(4\sin^2\theta + 9\cos^2\theta) = 36$ giving

$$(CA)^{-2}+(CB)^{-2}=\frac{13}{36}$$



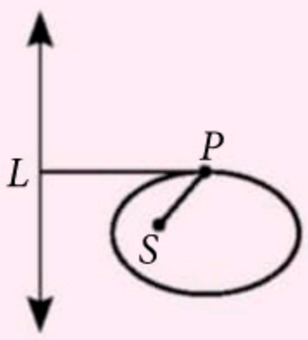
CONCEPT MAP

ELLIPSE

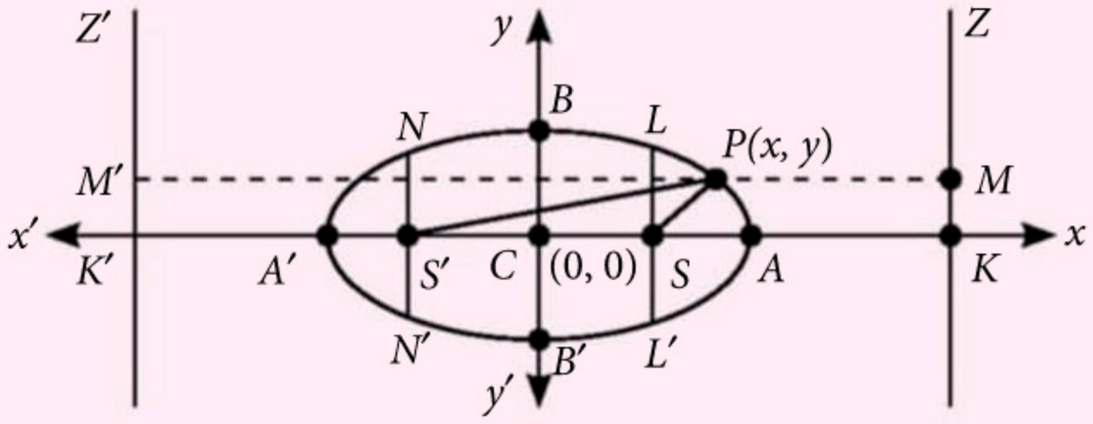
Class XI

An ellipse is the locus of a point which moves in such a way that the sum of its distances from two fixed points (foci) is always constant and is equal to the length of major axis of the ellipse.

It is locus of a point whose ratio of distances from a fixed point (focus) and a fixed line (directrix) is a constant less than 1.



$\frac{PS}{PL}$ = Eccentricity = $e < 1$



Standard equation of an ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a > b$.

- $C(0, 0)$ is the centre of the ellipse.
- Vertices are $A(a, 0)$, $A'(-a, 0)$.
- Line joining the foci S and S' is called axis of the ellipse.
- The distance $AA' = 2a$ = length of major axis.
- The distance $BB' = 2b$ = length of minor axis.
- The points $S(ae, 0)$, $S'(-ae, 0)$ are foci of the ellipse.
- ZK and $Z'K'$ are two directrices of the ellipse and $x = \pm a/e$ are the equations of directrices.
- Eccentricity, $e = \sqrt{\left(1-\frac{b^2}{a^2}\right)}$.
- Length of latus rectum, $LL' = 2SL = 2SL' = 2b^2/a$.

Definitions

General Terms

ELLIPSE

Conditions of Tangency for a Line to the Ellipse

Some Important Facts

If $S \equiv \frac{x^2}{a^2}+\frac{y^2}{b^2}-1$, $T \equiv \frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1$ and $S_1 \equiv \frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$, then

- Tangent to $S = 0$ at $P(x_1, y_1) : T = 0$
- Chord of $S = 0$ with midpoint $P(x_1, y_1) : T = S_1$
- Pair of tangents to $S = 0$ drawn from $P(x_1, y_1) : SS_1 = T^2$
- Chord of contact of tangents drawn from $P(x_1, y_1)$ to $S = 0 : T = 0$
- Normal to $S = 0$ at $P(x_1, y_1) : \frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$
- Auxiliary circle of $S = 0 : x^2 + y^2 = a^2$
- Director circle of $S = 0 : x^2 + y^2 = a^2 + b^2$
- Length of subtangent and subnormal at $P(x_1, y_1)$ of $S = 0$ are $\frac{a^2}{x_1}-x_1$ and $x_1(1 - e^2)$ respectively.

Ellipse	Equation of Line	Condition
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a > b)$	$y = mx + c$	$c^2 = a^2m^2 + b^2$
	$x\cos\alpha + y\sin\alpha = p$	$p^2 = a^2\cos^2\alpha + b^2\sin^2\alpha$
	$lx + my + n = 0$	$n^2 = a^2l^2 + b^2m^2$

PLANE

CONCEP MAP

Class XII

DEFINITION

A plane is a surface in which the line segment joining any two points completely lies on it.

EQUATION OF PLANE IN DIFFERENT FORMS

Equation	Vector form	Cartesian form
Equation of a plane in normal form	$\vec{r}\cdot\hat{n}=d$	$lx+my+nz=d$ where, l, m, n are direction cosines of $\hat{n}$.
Equation of a plane perpendicular to a given vector and passing through a given point	Let $\vec{a}$ be the position vector of given point P on the plane, then $(\vec{r}-\vec{a})\cdot\vec{n}=0$.	$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$, where a, b, c are dr's of $\vec{n}$ and $P(x_1, y_1, z_1)$ is the given point.
Equation of a plane passing through three non-collinear points	$(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0$	$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$
Equation of the plane passing through the intersection of two given planes	$\vec{r}\cdot(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$	$(a_1x+b_1y+c_1z+d_1)+\lambda(a_2x+b_2y+c_2z+d_2)=0$
Equation of a plane in intercept form		$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Line and Plane

- **Angle between line and plane :** The angle between a line and plane is the complement of the angle between the line and normal to the plane.

  Let line is $\vec{r}=\vec{a}+\mu\vec{b}$ and equation of plane is $\vec{r}\cdot\vec{n}=d$

  $\therefore \quad \sin\theta=\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}$, where θ is the angle between the line and the plane.

- **Image of a point in a plane :** Let $ax+by+cz+d=0$ be equation of plane and $P(x_1, y_1, z_1)$ be given point, say $P'(x', y', z')$ be the image of (x_1, y_1, z_1) in the given plane, then x', y', z' can be calculated as

$$\frac{x'-x_1}{a}=\frac{y'-y_1}{b}=\frac{z'-z_1}{c}=\frac{-2|ax_1+by_1+cz_1+d|}{a^2+b^2+c^2}.$$

Angle Between Two Planes

- **Vector form :** If $\vec{r}\cdot\vec{n}_1=d_1$ and $\vec{r}\cdot\vec{n}_2=d$ two planes and θ be the angle between t

  two planes, then $\cos\theta=\pm\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$.

- **Cartesian form :** If $a_1x+b_1y+c_1z+d$ and $a_2x+b_2y+c_2z+d_2=0$ are two pla

  then $\cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right.$

- Planes are perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$ or $\vec{n}_1\cdot\vec{n}_2=0$.

- Planes are parallel if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ or $\vec{n}_1=$

Distance of a Point from a Plan

- **Vector form :** Perpendicular distance of a $P(\vec{a})$ from the plane $\vec{r}\cdot\vec{n}=d$ is $p=\frac{|\vec{a}\cdot\vec{n}}{|\vec{n}}$

- **Cartesian form :** Perpendicular distance point (x_1, y_1, z_1) from the plane $ax+by+$ $=0$ is $p=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$.

Distance Between Two Parallel Pla

If $\vec{r}\cdot\vec{n}=d_1$ and $\vec{r}\cdot\vec{n}=d_2$ be two parallel pla

then $d=\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$, where $\vec{n}=a\hat{i}+b\hat{j}+$

Equation of Planes Bisecting the Angles Between Two Planes

- **Vector form :** If $\vec{r}\cdot\vec{n}_1=d_1$ and $\vec{r}\cdot\vec{n}_2=$ then $\frac{|\vec{r}\cdot\vec{n}_1-d_1|}{|\vec{n}_1|}=\frac{|\vec{r}\cdot\vec{n}_2-d_2|}{|\vec{n}_2|}$.

- **Cartesian form :** If $a_1x+b_1y+c_1z+d$ and $a_2x+b_2y+c_2z+d_2=0$, then

$$\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}=\pm\frac{a_1x+b_1y+c_1z+}{\sqrt{a_1^2+b_1^2+c_1^2}}$$

58. (d)

59. (c): Here, $t=\dfrac{2\sqrt{2}-(1+\sqrt{3})}{\sqrt{3}-1}$

$=\dfrac{\{2\sqrt{2}-(1+\sqrt{3})\}(\sqrt{3}+1)}{3-1}=\dfrac{2\sqrt{6}+2\sqrt{2}-4-2\sqrt{3}}{2}$

$=\sqrt{6}+\sqrt{2}-2-\sqrt{3}\ =\tan 7\dfrac{1^\circ}{2}$

Now, $f(x)=\dfrac{2x}{1-x^2}\ \Rightarrow\ f(t)=\dfrac{2t}{1-t^2}=\tan 15^\circ=2-\sqrt{3}$

$g(x)=\dfrac{3x-x^3}{1-3x^2}\Rightarrow\ g(t)=\dfrac{3t-t^3}{1-3t^2}=\tan 22\dfrac{1^\circ}{2}=\sqrt{2}-1$

Now, $f(g(t))=\tan 2\left(22\dfrac{1^\circ}{2}\right)=\tan 45^\circ=1$

$\therefore\quad (f(g(t)))'=0.$

60. (c): We know $-\dfrac{\pi}{2}\le\sin^{-1}x\le\dfrac{\pi}{2}$

$\therefore\ \sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\dfrac{3\pi}{2}$

$\Rightarrow\ \sin^{-1}x=\sin^{-1}y=\sin^{-1}z=\dfrac{\pi}{2}\ \Rightarrow\ x=y=z=1$

Also, $f(p+q)=f(p)\cdot f(q)\ \forall\ p, q\in R$ ...(i)

Given, $f(1)=1$

From (i), $f(1+1)=f(1)\cdot f(1)\ \Rightarrow\ f(2)=1^2=1$ ...(ii)

From (ii), $f(2+1)=f(2)\cdot(1)\ \Rightarrow f(3)=1^2\cdot 1=1^3=1$

Now, the given expression = 3 – 1 = 2

61. (d) : Here, $f(x)=\int_{x^2}^{x^3}\dfrac{1}{\log t}dt$

$\therefore\ f'(x)=\dfrac{3x^2}{3\log x}-\dfrac{2x}{2\log x}=\dfrac{x^2-x}{\log x}=\dfrac{x(x-1)}{\log x}$

For extremum, we put $f'(x)=0\Rightarrow x(x-1)=0$

$\therefore$ Critical points are $x=0$ and $x=1$

Let us consider $x=1$ as $x>0$

Now, for $x<1$, sign of $f'(x)$ = (+ve) (–ve) (–ve) > 0

and for $x>1$, sign of $f'(x)$ = (+ve) (+ve) (+ve) > 0

$\therefore$ $f'(x)$ does not change its sign in the immediate neighbourhood of $x=1$. So $x=1$ is neither the point of maxima nor minima.

62. (b) **63. (a)**

64. (c): Let P coordinates of be (α, β) then $PQ = 2\beta$ and $OP=\sqrt{\alpha^2+\beta^2}$

Since OPQ is an equilateral triangle, $OP = PQ$

$\Rightarrow\ \alpha^2+\beta^2=4\beta^2=\alpha^2=3\beta^2\ \Rightarrow\ \alpha=\pm\sqrt{3}\beta$

$\because$ (α, β) is situated on the hyperbola

$\therefore\quad \dfrac{\alpha^2}{a^2}-\dfrac{\beta^2}{b^2}=1\Rightarrow\dfrac{3\beta^2}{a^2}-\dfrac{\beta^2}{b^2}=1\ \Rightarrow\ \dfrac{3}{a^2}-\dfrac{1}{b^2}=\dfrac{1}{\beta^2}>0$

$\Rightarrow\ \dfrac{b^2}{a^2}>\dfrac{1}{3}\Rightarrow e^2-1>\dfrac{1}{3}\ \Rightarrow\ e^2>\dfrac{4}{3}\Rightarrow e>\dfrac{2}{\sqrt{3}}$

65. (d)

66. (a, b, c) : Here, tan$A$, tan$B$ are the roots of the equation $abx^2 - c^2x + ab = 0$

Therefore, $\tan A+\tan B=\dfrac{c^2}{ab}$ and $\tan A\tan B=1$

Now, tan$A$ tan$B$ = 1 $\Rightarrow$ cos$(A + B)$ = 0 ($\because$ A, B are acute)

$\therefore\quad A+B=\dfrac{\pi}{2}$ *i.e.*, $C=\dfrac{\pi}{2}$

Also, cot C = cot $\dfrac{\pi}{2}$ = 0

So, ΔABC is right angled at C and sin$A$ = cos$B \Rightarrow \sin^2A + \sin^2B = 1$

Also, $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}=\dfrac{\frac{a}{b}-\frac{b}{a}}{1+1}=\dfrac{a^2-b^2}{2ab}.$

67. (a,b) : $f(x) = [x^2] + [x] + 1 - 3 = \{[x] + 2]\}\{[x] - 1\}$

So, $\forall\ x\in[-2, -1)\cup[1, 2), f(x) = 0$

68. (b)

69. (a, b, c) : Let $p(n) = x(x^{n-1} - na^{n-1}) + a^n(n - 1)$. If $n = 1$, then

$p(n) = x(1 - 1) + (1 - 1)a$

$= x \cdot 0 + 0 \cdot a = 0$, which is divisible by $(x - a)^2$.

If $n = 2$, then $p(n) = x(x - 2a) + a^2(2 - 1)$

$= x^2 - 2xa + a^2 = (x - a)^2$, which is divisible by $(x - a)^2$.

If $n = 3$, then $p(n) = x(x^2 - 3a^2) + 2a^3$

$= x^3 - 3a^2x + 2a^3 = (x - a)(x^2 + ax - 2a^2)$

$= (x - a)(x - a)(x + 2a) = (x - a)^2(x + 2a)$,

which is divisible by $(x - a)^2$.

70. (a, b, c) **71. (a)** **72. (a, b, c)**

73. (a, b, c) : Let $z_1 = x_1; z_2, z_3 = x_2 \pm iy_2$

Now, $z_1+z_2+z_3=-\dfrac{b}{a}\ \Rightarrow\ x_1+2x_2=-\dfrac{b}{a}<0\Rightarrow ab>0$

Also, $z_1\,z_2\,z_3=x_1\left[x_2^2+y_2^2\right]=-\dfrac{d}{a}<0\Rightarrow ad>0$

Also, $2x_1x_2+x_2^2+y_2^2=\dfrac{c}{a}>0\Rightarrow bc>0$

74. (a, b, c, d) : As a_{912}, a_{951} and a_{480} are divisible by 3, none of them is prime, and for a_{91}, we have

$a_{91}=\dfrac{10^{91}-1}{10-1}=\dfrac{10^{91}-1}{10^7-1}\times\dfrac{10^7-1}{10-1}$

$= (1 + 10^7 + ... + 10^{84})\ (1 + 10 + ... + 10^6)$

$\Rightarrow$ a_{91} is not prime.

75. (b, d) : $f(x) = 2 - \sin^{-1}x$ if $-1 \le x \le 0$

$2 + \sin^{-1}x$ if $0 < x \le 1$

Hence, f is continuous everywhere on the domain

$f'(x)=\dfrac{-1}{\sqrt{1-x^2}}$ if $-1<x<0$, $f'(x)=\dfrac{1}{\sqrt{1-x^2}}$ if $0<x<1$

$\therefore$ f is not differentiable at $x = 0$.

CONCEPT BOOSTER

Integration

Class XII

*ALOK KUMAR

DEFINITION

If $F(x)$ be a differentiable function of x such that $\frac{d}{dx}[F(x)] = f(x)$. Then $F(x)$ is called the integral or anti-derivative or primitive function of the integrand $f(x)$. Symbolically, it is written as $\int f(x)dx = F(x) + c$, where c is called the constant of integration and can take any real value.

BASIC FORMULAE

Anti-derivatives or integrals of some of the widely used functions (integrands) are given below:

- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$
- $\int \frac{1}{x}dx = \ln|x| + c$
- $\int e^x\,dx = e^x + c$
- $\int a^x\,dx = \frac{a^x}{\ln a} + c, a > 0, a \neq 1$
- $\int \cos x\,dx = \sin x + c$
- $\int \sin x\,dx = -\cos x + c$
- $\int \sec^2 x\,dx = \tan x + c$
- $\int \sec x \tan x\,dx = \sec x + c$
- $\int \text{cosec}x \cot x\,dx = -\text{cosec}x + c$
- $\int \text{cosec}^2 x\,dx = -\cot x + c$
- $\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{1}{|x|\sqrt{x^2 - 1}}dx = \sec^{-1} x + c$
- $\int \cot x\,dx = \ln|\sin x| + c$
- $\int \tan x\,dx = \ln|\sec x| + c$
- $\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\int \text{cosec}x\,dx = \ln|\cot x - \text{cosec}\,x| + c$

STANDARD FORMULAE

- $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$
- $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\ln\left|x + \sqrt{x^2 + a^2}\right| + c$
- $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left|x + \sqrt{x^2 - a^2}\right| + c$
- $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left|x + \sqrt{x^2 - a^2}\right| + c$
- $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left|x + \sqrt{x^2 + a^2}\right| + c$
- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x - a}{x + a}\right| + c$

METHODS OF INTEGRATION

In order to find the integral of complex problems, generally three rules of integration are used :

- Integration by substitution.
- Integration by parts.
- Integration by partial fractions.

INTEGRATION BY SUBSTITUTION

Direct Substitution

- If integral is of the form $\int f(g(x))\,g'(x)\,dx$, then put $g(x) = t$, provided $\int f(t)dt$ exists.

*Alok Kumar, a B.Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

- $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$
- $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c$

Standard Substitutions

Expression	Substitution
$x^2 + a^2$ or $\sqrt{x^2 + a^2}$	$x = a \tan\theta$ or $a \cot\theta$
$x^2 - a^2$ or $\sqrt{x^2 - a^2}$	$x = a \sec\theta$ or $a \operatorname{cosec}\theta$
$a^2 - x^2$ or $\sqrt{a^2 - x^2}$	$x = a \sin\theta$ or $x = a \cos\theta$
$\sqrt{a+x}$ or $\sqrt{a-x}$	$x = a \cos 2\theta$
$\left(x \pm \sqrt{x^2 \pm a^2}\right)^n$	expression inside the bracket = t
$\sqrt{(x-a)(b-x)}$	$x = a\cos^2\theta + b\sin^2\theta$
$\frac{1}{(x+b)^{1-\frac{1}{n}}(x+a)^{1+\frac{1}{n}}}\ (n \in N, n>1)$	$\frac{x+b}{x+a} = t$

Indirect Substitution

If the integrand is of the form $f(x)\ g(x)$, where $g(x)$ is a function of the integral of $f(x)$, then put integral of $f(x) = t$.

INTEGRATION BY PARTS

- If u and v be two functions of x, then integral of product of these two functions is given by

$$\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$$

- In applying the above rule care has to be taken in the selection of the first function (u) and the second function (v). Normally we use the following methods:
- If in the product of the two functions, one of the functions is not directly integrable (*e.g.*, $\ln x$, $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ etc.), then we take it as the first function and the remaining function is taken as the second function.
- If there is no other function, then unity is taken as the second function.
- If both the functions are directly integrable, then the first function is chosen in such a way that the derivative of the function thus obtained under integral sign is easily integrable. Usually we use the following preference order for the first function.
(Inverse, Logarithmic, Algebraic, Trigonometric, Exponent)
In the above stated order, the function on the left is always chosen as the first function. This rule is called as ILATE. For example, in $\int x \sin x\,dx$, x is taken as the first function and $\sin x$ is taken as the second function.

INTEGRATION BY PARTIAL FRACTIONS

- A function of the form $P(x)/Q(x)$, where $P(x)$ and $Q(x)$ are polynomials, is called a rational function. Consider the rational function

$$\frac{x+7}{(2x-3)(3x+4)} = \frac{1}{2x-3} - \frac{1}{3x+4}$$

- The two fractions on the RHS are called partial fractions. To integrate the rational function on the LHS, it is enough to integrate the two fractions on the RHS, which are easily integrable. This is known as method of partial fractions. In case the degree of $P(x)$ (numerator) is not less than that of $Q(x)$ (denominator), we carry out the division of $P(x)$ by $Q(x)$ and reduce the degree of the numerator.
- In order to write $P(x)/Q(x)$ in partial fractions, first of all we write
$Q(x) = (x-a)^k \ldots (x^2 + \alpha x + \beta)^r \ldots$ where binomials are different, and then set

$$\frac{P(x)}{Q(x)} = \frac{A_1}{(x-a)} + \frac{A_2}{(x-a)^2} + \ldots + \frac{A_k}{(x-a)^k}$$
$$+ \frac{M_1x + N_1}{x^2+\alpha x+\beta} + \frac{M_2x+N_2}{(x^2+\alpha x+\beta)^2} + \ldots + \frac{M_rx+N_r}{(x^2+\alpha x+\beta)^r} + \ldots,$$

where $A_1, A_2, \ldots, A_k, M_1, M_2, \ldots\ldots, M_r, N_1, N_2, \ldots\ldots, N_r$ are real constants to be determined. These are determined by reducing both sides of the above identity to integral form and equating the coefficients of equal powers of x, which gives a system of linear equations in the coefficient. The constants can also be obtained by substituting suitably chosen numerical values of x in both sides of the identity.
- Before proceeding to write a rational function as a sum of partial fractions, we should be taken to ascertain that it is either a proper rational fraction or is rewritten as one.

ALGEBRAIC INTEGRALS

- A rational function $\frac{P(x)}{Q(x)}$ is proper if the degree of polynomial $Q(x)$ is greater than the degree of the polynomial $P(x)$. In case the degree of $P(x)$ is greater than or equal to the degree of $Q(x)$, we first write $\frac{P(x)}{Q(x)} = h(x) + \frac{P_1(x)}{Q(x)}$, where $h(x)$ is a polynomial and $P_1(x)$ is a polynomial of degree less than the degree of polynomial $Q(x)$.

- **Integrals of the type** $\int \frac{px+q}{(ax^2+bx+c)}dx$,

$\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$, $\int (px+q)\sqrt{ax^2+bx+c}\,dx$:

In these type of integrals, we write $px + q = l$ (differential coefficient of $ax^2 + bx + c$) + m ...(i) Find values of l and m by comparing the coefficients of x and constant term on both sides of (i). In this way, the integration will reduce to the sum of two integrals which can be integrated easily.

- **Integrals of the type**

$$\frac{ax^2+bx+c}{px^2+qx+r} \text{ or } \frac{ax^2+bx+c}{\sqrt{px^2+qx+r}}:$$

In this case substitute $ax^2 + bx + c = M(px^2 + qx + r) + N(2px + q) + R$. M, N and R can be calculated by comparing the coefficients. Then, integration will reduce to integration of three independent functions.

Integration of Irrational Algebraic Fractions

- Irrational functions of $(ax + b)^{1/n}$ and x can be easily evaluated by the substitution, $t^n = ax + b$. Thus,

$$\int f(x,(ax+b)^{1/n})\,dx = \int f\left(\frac{t^n-b}{a}, t\right)\frac{nt^{n-1}}{a}dt$$

- To integrate $\int \frac{dx}{(Ax^2+B)\sqrt{Cx^2+D}}$, we put $x = 1/t$, which will reduce the given integral to

$$-\int \frac{tdt}{(A+Bt^2)\sqrt{C+Dt^2}}$$

Now, substitute $C + Dt^2 = u^2$, which will reduce it to the form $\int \frac{du}{u^2 \pm a^2}$

- To integrate $\int \frac{(ax^2+bx+c)dx}{(dx+e)\sqrt{fx^2+gx+h}}$, we write, $ax^2 + bx + c = A_1(dx + e)(2fx + g) + B_1(dx + e) + C_1$, where A_1, B_1 and C_1 can be determined.

TRIGONOMETRIC INTEGRALS

Integrals of the form $\int R(\sin x, \cos x)\,dx$

- Here R is a rational function of $\sin x$ and $\cos x$. In this case we take universal substitution $\tan(x/2) = t$.

So, $\sin x = \frac{2t}{1+t^2}$; $\cos x = \frac{1-t^2}{1+t^2}$

Integrals of the form

$$\int \frac{p\cos x + q\sin x + r}{a\cos x + b\sin x + c}dx \text{ or } \int \frac{p\cos x + q\sin x}{a\cos x + b\sin x}dx$$

- In this integral express numerator as l (denominator) + m(derivative of denominator) + n and find the values of l, m, n by comparing the coefficients of $\sin x$, $\cos x$ and constant term and split the integral into sum of three integrals as :

$$l\int dx + m\int \frac{\text{derivative of denominator}}{\text{denominator}}dx + n\int \frac{dx}{\text{denominator}}$$

- Express numerator as l (denominator) + m(derivative of denominator) and find the values of l and m as above.

Integrals of the form $\int (\sin^m x \cos^n x)\,dx$, where m and $n \in N$

- If one of m and n is odd, then substitute for term of even power.
- If both m and n are odd, substitute either of the term.
- If both m and n are even, use trigonometric identities only.

DEFINITE INTEGRATION

Fundamental Theorem of Integral Calculus (Newton-Leibnitz Formula)

- If $f(x)$ is a continuous function on $[a, b]$ and $F(x)$ is an anti derivative of $f(x)$ on $[a, b]$ *i.e.*,

$F'(x) = f(x)\ \forall\ x \in (a, b)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

Here, a and b are the lower and the upper limits of integration.

Properties of Definite Integration

- $\int_a^b f(x)\,dx = \int_a^b f(t)dt$

- $\int_a^b f(x)\,dx = -\int_b^a f(x)dx$
- $\int_a^b f(x)\,dx = \int_a^c f(x)dx + \int_c^b f(x)dx, a<c<b$
- $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)dx$
- $\int_0^a f(x)\,dx = \int_0^{a/2}[f(x)+f(a-x)]dx$

  Also, If $f(x)=f(a-x)$, then $\int_0^a f(x)dx = 2\int_0^{a/2} f(x)dx$

  If $f(x) = -f(a-x)$, then $\int_0^a f(x)dx = 0$
- $\int_{-a}^a f(x)dx = \int_0^a [f(x)+f(-x)]dx$

  Also, $\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is even.} \\ 0, & \text{if } f(x) \text{ is odd.} \end{cases}$
- $\int_a^b f(x)dx = (b-a)\int_0^1 f((b-a)x+a)dx$
- If $f(x)$ is a periodic function with period T, then

  $\int_a^{a+nT} f(x)dx = n\int_0^T f(x)dx$ where $n \in I$,

Differentiation under the Integral Sign (Leibnitz's Rule)

- If g is continuous on $[a, b]$ and $f_1(x)$ and $f_2(x)$ are differentiable functions whose values lie in $[a, b]$, then

  $\frac{d}{dx}\int_{f_1(x)}^{f_2(x)} g(t)dt = g(f_2(x))f_2'(x) - g(f_1(x))f_1'(x)$

Definite Integral as Limit of a Sum

- $\int_0^1 f(x)\,dx = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$

PROBLEMS

Single Correct Answer Type

1. $\int \frac{dx}{\sqrt{(x-a)(b-x)}} =$

(a) $2\sin^{-1}\sqrt{\left(\frac{x-a}{b-a}\right)}+c$ (b) $\sin^{-1}\sqrt{\left(\frac{x-a}{b-a}\right)}+c$

(c) $2\sin^{-1}\sqrt{\left(\frac{x+a}{b-a}\right)}+c$ (d) none of these

2. If $\int x e^x \cos x\,dx = f(x)+c$, then $f(x)$ is equal to

(a) $\frac{e^x}{-2}\{(1-x)\sin x - x\cos x\}$

(b) $\frac{e^x}{2}\{(1-x)\sin x + x\cos x\}$

(c) $\frac{e^x}{2}\{(1+x)\sin x - x\cos x\}$ (d) none of these

3. If $I = \int \frac{dx}{e^x+4e^{-x}} = f(x)+c$, then $f(x)$ equals to

(a) $2\tan^{-1}(2e^x)$ (b) $\frac{1}{2}\tan^{-1}\left(\frac{e^x}{2}\right)$

(c) $2\tan^{-1}\frac{e^x}{2}$ (d) $\frac{1}{2}\tan^{-1}(2e^{2x})$

4. $\int \frac{\sin x}{\sin x - \cos x}dx$ is equal to

(a) $\frac{x}{2}-\frac{1}{2}\log(\sin x-\cos x)+c$

(b) $\frac{x}{2}+\frac{1}{2}\log(\sin x-\cos x)+c$

(c) $\frac{x}{2}+\frac{1}{2}\log(\sin x+\cos x)+c$

(d) none of these

5. $\int \frac{(1+x)}{x(1+xe^x)^2}dx$ is equal to

(a) $\ln\left(\frac{xe^x}{1+xe^x}\right)-\left(\frac{1}{1+xe^x}\right)+c$

(b) $\ln\left(\frac{xe^x}{1+xe^x}\right)+\left(\frac{1}{1-xe^x}\right)+c$

(c) $\ln\left(\frac{xe^x}{1-xe^x}\right)+\left(\frac{1}{1+xe^x}\right)+c$

(d) $\ln\left(\frac{xe^x}{1+xe^x}\right)+\left(\frac{1}{1+xe^x}\right)+c$

6. $\int \frac{\cos 5x+\cos 4x}{1-2\cos 3x}dx$ is equal to

(a) $\frac{\sin 2x}{2}-\sin x+c$ (b) $-\frac{\sin 2x}{2}+\sin x+c$

(c) $-\frac{\sin 2x}{2}-\sin x+c$ (d) $\frac{\sin 2x}{2}+\sin x+c$

7. $\int \frac{\sec x \operatorname{cosec} x}{\log \tan x} dx$ is equal to

(a) $\log(\tan x) + c$ (b) $\cot(\log x) + c$

(c) $\log \log(\tan x) + c$ (d) $\tan(\log x) + c$

8. $\int \frac{\left(x^4 - x\right)^{\frac{1}{4}}}{x^5} dx$ is equal to

(a) $\frac{4}{15}\left(1-\frac{1}{x^3}\right)^{\frac{5}{4}} + c$ (b) $\frac{4}{5}\left(1-\frac{1}{x^3}\right)^{\frac{5}{4}} + c$

(c) $\frac{4}{15}\left(1+\frac{1}{x^3}\right)^{\frac{5}{4}} + c$ (d) none of these

9. $\int \frac{\log(x+1) - \log x}{x(x+1)} dx$ equals

(a) $-\log\left(\frac{x+1}{x}\right) + c$ (b) $-\log\left[\log\left(\frac{x+1}{x}\right)\right] + c$

(c) $-\frac{1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2 + c$

(d) $c - \frac{1}{2}[\log(x+1)^2 - (\log x)^2]$

10. $\int \frac{\sin x - \cos x}{\sqrt{1-\sin 2x}} e^{\sin x} \cos x dx$ is equal to

(a) $e^{\sin x} + c$ (b) $e^{\sin x - \cos x} + c$

(c) $e^{\sin x + \cos x} + c$ (d) $e^{\cos x - \sin x} + c$

11. $I = \int \frac{(1-\cos x)\, dx}{\cos x(1+\cos x)} =$

(a) $\log(\sec x + \tan x) - 2\tan\left(\frac{x}{2}\right) + c$

(b) $\log(\sec x + \tan x) + 2\tan\left(\frac{x}{2}\right) + c$

(c) $\log(\sec x - \tan x) - 2\tan\left(\frac{x}{2}\right) + c$

(d) none of these

12. $\int \{\sin(\log x) + \cos(\log x)\} dx$ is equal to

(a) $x \sin(\log x) + c$ (b) $x \cos(\log x) + c$

(c) $x \log(\sin x) + c$ (d) $x \log(\cos x) + c$

13. $\int \frac{dx}{\cos^6 x + \sin^6 x}$ is equal to

(a) $\log_e(\tan x - \cot x) + c$ (b) $\log_e(\cot x - \tan x) + c$

(c) $\tan^{-1}(\tan x - \cot x) + c$ (d) $2\tan^{-1}(2\cot 2x) + c$

14. $\int \frac{3 + 2\cos x}{(2 + 3\cos x)^2} dx$ is equal to

(a) $\left(\frac{\sin x}{3\cos x + 2}\right) + c$ (b) $\left(\frac{2\cos x}{3\sin x + 2}\right) + c$

(c) $\left(\frac{2\cos x}{3\cos x + 2}\right) + c$ (d) $\left(\frac{2\sin x}{3\sin x + 2}\right) + c$

15. $\int \sin(\ln x) dx$ is equal to

(a) $\frac{x}{2}[\sin(\ln x) + \cos(\ln x)] + c$

(b) $\frac{x}{2}[\cos(\ln x) - \sin(\ln x)] + c$

(c) $\frac{x}{2}[\sin(\ln x) - \cos(\ln x)] + c$

(d) $x[\sin(\ln x) - \cos(\ln x)] + c$

16. $\int \frac{1}{x[(\ln x)^2 + 4\ln x - 1]} dx$ is equal to

(a) $\frac{1}{\sqrt{5}} \ln\left|\frac{\ln x + 2 - \sqrt{5}}{\ln x + 2 + \sqrt{5}}\right| + c$ (b) $\frac{1}{2} \ln\left|\frac{\ln x + 2 - \sqrt{5}}{\ln x + 2 + \sqrt{5}}\right| + c$

(c) $\frac{1}{2\sqrt{5}} \ln\left|\frac{\ln x + 2 - \sqrt{5}}{\ln x - 2 + \sqrt{5}}\right| + c$

(d) $\frac{1}{2\sqrt{5}} \ln\left|\frac{\ln x + 2 - \sqrt{5}}{\ln x + 2 + \sqrt{5}}\right| + c$

17. $\int \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$ is equal to

(a) $\frac{1}{40} \ln\left|\frac{5 + 4(\sin x - \cos x)}{5 - 4(\sin x - \cos x)}\right| + c$

(b) $\frac{1}{40} \ln\left|\frac{5 - 4(\sin x - \cos x)}{5 + 4(\sin x - \cos x)}\right| + c$

(c) $\frac{1}{40} \ln\left|\frac{5 + 4(\sin x + \cos x)}{5 - 4(\sin x + \cos x)}\right| + c$

(d) $\frac{1}{40} \ln\left|\frac{5 - 4(\sin x + \cos x)}{5 + 4(\sin x - \cos x)}\right| + c$

18. For what values of a and b, the equation $\int (\sin 2x - \cos 2x) dx = \frac{1}{\sqrt{2}} \sin(2x - a) + b$ holds good?

(a) $a = -\frac{5\pi}{4}$, b is any arbitrary constant

(b) $a = \frac{5\pi}{4}$, b is any arbitrary constant

(c) $a = -\frac{\pi}{4}$, b is any arbitrary constant

(d) $a = \frac{\pi}{4}$, b is any arbitrary constant

19. $\int \cos\left(\ln\frac{x}{a}\right)dx$ is equal to

(a) $x\left[\cos\left(\ln\frac{x}{a}\right)-\sin\left(\ln\frac{x}{a}\right)\right]+c$

(b) $\frac{x}{2}\left[\cos\left(\ln\frac{x}{a}\right)+\sin\left(\ln\frac{x}{a}\right)\right]+c$

(c) $\frac{x}{2}\left[\cos\left(\ln\frac{x}{a}\right)-\sin\left(\ln\frac{x}{a}\right)\right]+c$

(d) $x\left[\cos\left(\ln\frac{x}{a}\right)+\sin\left(\ln\frac{x}{a}\right)\right]+c$

20. $\int \frac{x\cos x}{(x\sin x+\cos x)^2}dx$ is equal to

(a) $\frac{1}{(x\sin x+\cos x)^2}+c$ (b) $-\frac{1}{(x\sin x+\cos x)}+c$

(c) $\frac{1}{(x\sin x+\cos x)^3}+c$ (d) $\frac{1}{(x\sin x+\cos x)^4}+c$

21. $\int \frac{(\sqrt{x^2+1})\{\ln(x^2+1)-2\ln x\}}{x^4}dx$ is equal to

(a) $\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(b) $\frac{1}{9}\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2+3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(c) $\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2+3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(d) $\frac{1}{9}\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

22. $\int \frac{dx}{5+4\cos x}$ is equal to

(a) $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+c$ (b) $-\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+c$

(c) $\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+c$ (d) none of these

23. If $I_n=\int(\ln x)^n dx$, then I_n+nI_{n-1} is equal to

(a) $x(\ln x)^{n-1}$ (b) $x(\ln x)^n$

(c) $nx(\ln x)^n$ (d) none of these

Multiple Correct Answer Type

24. Let $G(x,t)=\begin{cases}x(t-1), & \text{where } x\le t\\ t(x-1), & \text{where } t<x\end{cases}$ and t is continuous function of x in $[0, 1]$.

If $g(x)=\int_0^1 f(t)G(x,t)dt$, then

(a) $g(0)=1$ (b) $g(0)=0$

(c) $g(1)=1$ (d) $g''(x)=f(x)$

25. Let $T>0$ be a fixed real number. Suppose $f(x)$ is a continuous function for all $x\in R$, $f(x+T)=f(x)$. If $I=\int_0^T f(x)dx$, then

(a) $\int_5^{5+5T} f(x)dx=5I$ (b) $\int_5^{5+5T} f(2x)dx=10I$

(c) $\int_5^{5+5T} f(3x)dx=5I$ (d) $\int_5^{5+5T} f(3x)dx=15I$

26. If $I_1=\int_0^1 2^{x^2}dx$, $I_2=\int_0^1 2^{x^3}dx$, $I_3=\int_1^2 2^{x^2}dx$ and $I_4=\int_1^2 2^{x^3}dx$, then

(a) $I_1>I_2$ (b) $I_2>I_1$ (c) $I_3>I_4$ (d) $I_4>I_3$

27. The value of $\int_{1/e}^{\tan x}\frac{tdt}{1+t^2}+\int_{1/e}^{\cot x}\frac{dt}{t(1+t^2)}$ is

(a) $\frac{1}{2+\tan^2 x}$ (b) 1

(c) $\frac{\pi}{4}$ (d) $\frac{2}{\pi}\int_{-1}^{1}\frac{dt}{1+t^2}$

28. If $J=\int\frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx$, then J equals to

(a) $\frac{x^7}{2x^7+x^2+1}+c$ (b) $\frac{x^5}{x^2+1+2x^7}+c$

(c) $\frac{-1}{2x^7+x^2+1}+c$

(d) $\frac{p(x)}{q(x)}+c$, $\deg(p(x))=\deg(q(x))=7$

Comprehension Type

Paragraph for Question No. 29 to 30

Given: $\int_a^b f(x)d\alpha(x)+\int_a^b \alpha(x)df(x)=\alpha(b)f(b)-\alpha(a)f(a)$

29. $\int_0^3 (x^2+1)d[x]$ (where $[\cdot]$ denotes the greatest integer function) is equal to

(a) 3 (b) $\frac{9}{2}$ (c) 17 (d) $\frac{27}{2}$

30. $\int_0^6 (x^2+[x])d|3-x|$ (where $[\cdot]$ denotes the greatest integer function) is equal to

(a) 36 (b) 72 (c) 63 (d) 126

Paragraph for Question No. 31 to 33

Let n be a non-negative integer and, let

$I_n = \int x^n \sqrt{a^2-x^2}\,dx (a>0)$ we can find relation among I_n, I_{n-1}, I_{n-2}. It can be observed that I_1 is elementary integration whose value is $-\frac{1}{3}(a^2-x^2)^{3/2}$.

If $I_n = -\frac{x^{n-1}(a^2-x^2)^{3/2}}{A} + a^2 B I_{n-2}$, where A and B are constants. Then

31. A must be equal to

(a) $n+1$ (b) $n-1$ (c) $n+2$ (d) n

32. B must be equal to

(a) $\frac{n+1}{n+2}$ (b) $\frac{n+2}{n+1}$ (c) $\frac{n}{n+2}$ (d) $\frac{n-1}{n+2}$

33. The value of $\int_0^a x^4\sqrt{a^2-x^2}\,dx$ must be

(a) $\frac{\pi a^6}{32}$ (b) $\frac{\pi a^4}{16}$ (c) $\frac{\pi a^4}{64}$ (d) $\frac{\pi a^6}{16}$

Matrix-Match Type

34. Match the following

Column-I		Column-II	
A.	$\int \sqrt{\text{cosec}\,x+1}\,dx =$	p.	$\frac{1}{\sqrt{2}}\cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right)+c$
B.	$\int \frac{\sin x}{\sqrt{3-\cos 2x}}dx =$	q.	$\sin^{-1}(\sin x\cos x)+c$
C.	$\int \frac{\cos x}{\sqrt{\cos 2x}}dx =$	r.	$\frac{-1}{\sqrt{2}}\sin^{-1}(\sqrt{\cos 2x})+c$
D.	$\int \frac{\cos 2x}{\sqrt{\cos^2 x+\sin^4 x}}dx =$	s.	$\sin^{-1}(2\sin x-1)+c$

35. Match the anti-derivative given in column-II of the functions given in column-I.

Column-I		Column-II	
A.	$\frac{\sec x}{(\sec x+\tan x)^2}$	p.	$\log\left\|\frac{\sin x-2}{\sin x-1}\right\|+c$
B.	$\frac{\cos x}{(\sin x-1)(\sin x-2)}$	q.	$-\frac{\cos^2 x}{2(1+\sin x)^2}+c$
C.	$\sin^{-1}\left(\frac{2x}{1+x^2}\right), \|x\|<1$	r.	$2x\tan^{-1}x - \log(1+x^2)+c$
D.	$\sqrt{\tan x}+\sqrt{\cot x}$	s.	$\sqrt{2}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+c$

SOLUTIONS

1. (a) : Putting $x = a\cos^2\theta + b\sin^2\theta$ in the given integral, we get

$$\int \frac{2(b-a)\sin\theta\cos\theta d\theta}{\sqrt{(a\cos^2\theta+b\sin^2\theta-a)(b-a\cos^2\theta-b\sin^2\theta)}}$$

$$= \int \frac{2(b-a)\sin\theta\cos\theta d\theta}{(b-a)\sin\theta\cos\theta} = \int 2\,d\theta$$

$$= 2\theta + c = 2\sin^{-1}\sqrt{\left(\frac{x-a}{b-a}\right)}+c$$

2. (a) : Let I = real part of $\int x e^{(1+i)x}dx$

Now, $\int xe^{(1+i)x}\,dx = \frac{xe^{(1+i)x}}{1+i} - \int \frac{e^{(1+i)x}}{1+i}dx$

$$= \frac{xe^{(1+i)x}}{1+i} - \frac{e^{(1+i)x}}{(1+i)^2} = e^{(1+i)x}\left[\frac{x(1+i)-1}{(1+i)^2}\right]$$

$$= e^x(\cos x + i\sin x)\left[\frac{(x-1)+ix}{1+2i-1}\right]$$

$$= \frac{e^x}{-2}[i\cos x - \sin x][(x-1)+ix]$$

$$\therefore \quad I = -\frac{e^x}{2}[(1-x)\sin x - x\cos x]+c$$

3. (b)

4. (b) : Let $\sin x = A(\sin x - \cos x) + B(\cos x + \sin x)$

$\Rightarrow \sin x = (A+B)\sin x + (B-A)\cos x$

$\Rightarrow A + B = 1$ and $B - A = 0 \Rightarrow A = B = 1/2$

$$\therefore \quad I = \int \frac{\frac{1}{2}(\sin x-\cos x)+\frac{1}{2}(\cos x+\sin x)}{\sin x-\cos x}dx$$

$$= \frac{x}{2} + \frac{1}{2}\log(\sin x - \cos x)+c$$

5. (d) : Let $I=\int\frac{(1+x)}{x(1+xe^x)^2}dx=\int\frac{(1+x)e^x}{(xe^x)(1+xe^x)^2}dx$

Put $1+xe^x=p\Rightarrow e^x(1+x)dx=dp\ \therefore\ I=\int\frac{dp}{(p-1)p^2}$

Now let, $\frac{1}{(p-1)p^2}=\frac{A}{(p-1)}+\frac{B}{p}+\frac{C}{p^2}$

$\Rightarrow\quad 1=Ap^2+Bp\,(p-1)+C(p-1)$

For $p=1$, $p=0$ and $p=-1$, we get

$A=1$, $C=-1$ and $B=-1$

$\therefore\quad I=\int\frac{1}{(p-1)}dp-\int\frac{dp}{p}-\int\frac{dp}{p^2}=\ln\frac{(p-1)}{p}+\frac{1}{p}+c$

$=\ln\left(\frac{xe^x}{1+xe^x}\right)+\left(\frac{1}{1+xe^x}\right)+c$

6. (c) : We have, $\int\frac{\cos 5x+\cos 4x}{1-2\cos 3x}dx$

$=\int\frac{2\cos\frac{9x}{2}\cos\frac{x}{2}\cos\frac{3x}{2}}{\left[1-2\left(2\cos^2\frac{3x}{2}-1\right)\right]\cos\frac{3x}{2}}dx$

$=\int\frac{2\cos\frac{9x}{2}\cos\frac{3x}{2}\cos\frac{x}{2}}{3\cos\frac{3x}{2}-4\cos^2\frac{3x}{2}}dx=\int\frac{2\cos\frac{9x}{2}\cos\frac{3x}{2}\cos\frac{x}{2}}{-\cos\frac{9x}{2}}dx$

$=-\int(\cos 2x+\cos x)\,dx\ =-\frac{\sin 2x}{2}-\sin x+c$

7. (c) **8. (a)** **9. (c)** **10. (a)**

11. (a) : Given, $I=\int\frac{(1-\cos x)\,dx}{\cos x(1+\cos x)}$

$=\int\left(\frac{1}{\cos x}-\frac{2}{1+\cos x}\right)dx=\int\left(\sec x-\sec^2\frac{x}{2}\right)dx$

$=\log(\sec x+\tan x)-2\tan\left(\frac{x}{2}\right)+c$

12. (a)

13. (c) : Let $I=\int\frac{dx}{\cos^6 x+\sin^6 x}=\int\frac{\sec^6 x}{1+\tan^6 x}dx$

$=\int\frac{(1+\tan^2 x)^2\sec^2 x dx}{1+\tan^6 x}$

Put $\tan x=p\Rightarrow\sec^2 x\,dx=dp$

$\therefore\quad I=\int\frac{(1+p^2)^2dp}{1+p^6}=\int\frac{(1+p^2)}{p^4-p^2+1}dp$

$=\int\frac{p^2\left(1+\frac{1}{p^2}\right)}{p^2\left(p^2+\frac{1}{p^2}-1\right)}dp$

Put $p-\frac{1}{p}=k\Rightarrow\left(1+\frac{1}{p^2}\right)dp=dk$

$\therefore\quad I=\int\frac{dk}{k^2+1}=\tan^{-1}(k)+c$

$=\tan^{-1}\left(p-\frac{1}{p}\right)+c=\tan^{-1}(\tan x-\cot x)+c$

14. (a) : Let $I=\int\frac{3+2\cos x}{(2+3\cos x)^2}dx$

$=\int\frac{(3\,\mathrm{cosec}^2 x+2\cot x\ \mathrm{cosec}\,x)}{(2\mathrm{cosec}\,x+3\cot x)^2}dx$

$=-\int\frac{-3\mathrm{cosec}^2 x-2\cot x\ \mathrm{cosec}\,x}{(2\,\mathrm{cosec}\,x+3\cot x)^2}dx$

$=\frac{1}{2\,\mathrm{cosec}\,x+3\cot x}+c=\left(\frac{\sin x}{2+3\cos x}\right)+c$

15. (c)

16. (d) : Let $I=\int\frac{1}{x[(\ln x)^2+4\ln x-1]}dx$

Put $\ln x=t\quad\Rightarrow\ \frac{1}{x}dx=dt$

$\therefore\quad I=\int\frac{dt}{t^2+4t-1}=\int\frac{dt}{(t+2)^2-(\sqrt{5})^2}$

$=\frac{1}{2\sqrt{5}}\ln\left|\frac{t+2-\sqrt{5}}{t+2+\sqrt{5}}\right|+c=\frac{1}{2\sqrt{5}}\ln\left|\frac{\ln x+2-\sqrt{5}}{\ln x+2+\sqrt{5}}\right|+c$

17. (a) : Let $I=\int\frac{\sin x+\cos x}{9+16\sin 2x}dx$

Put $t=\sin x-\cos x\ \Rightarrow dt=(\sin x+\cos x)dx$

Also, $t^2=1-\sin 2x\Rightarrow\quad\sin 2x=1-t^2$

$\therefore\quad I=\int\frac{dt}{9+16(1-t^2)}=\int\frac{dt}{25-16t^2}$

$=\frac{1}{16}\int\frac{dt}{\left(\frac{5}{4}\right)^2-t^2}=\frac{1}{16}\times\frac{1}{2\left(\frac{5}{4}\right)}\ln\left|\frac{\frac{5}{4}+t}{\frac{5}{4}-t}\right|+c$

$=\frac{1}{40}\ln\left|\frac{5+4t}{5-4t}\right|+c=\frac{1}{40}\ln\left|\frac{5+4(\sin x-\cos x)}{5-4(\sin x-\cos x)}\right|+c$

18. (a) : We have, $\int(\sin 2x-\cos 2x)\,dx$

$$=-\int\sqrt{2}\cos\left(2x+\frac{\pi}{4}\right)dx$$

$$=\frac{-\sqrt{2}}{2}\sin\left(2x+\frac{\pi}{4}\right)+c=\frac{1}{\sqrt{2}}\sin\left(2x+\frac{5\pi}{4}\right)+c$$

$\therefore\ a=-\frac{5\pi}{4}$, b is any arbitrary constant.

19. (b) **20. (b)**

21. (d) : Let $I=\int\frac{\left(\sqrt{\frac{x^2+1}{x^2}}\right)\ln\left(\frac{x^2+1}{x^2}\right)}{x^3}dx$

Put $\frac{x^2+1}{x^2}=t\ \Rightarrow\ -\frac{2}{x^3}dx=dt$

$$\therefore\quad I=-\frac{1}{2}\int\sqrt{t}\ln t\,dt\ =-\frac{1}{2}\left[(\ln t).\frac{2t^{3/2}}{3}-\frac{2}{3}\int\frac{1}{t}.t^{3/2}dt\right]$$

$$=\frac{1}{9}t^{3/2}\,[2-3\ln t]+c$$

$$=\frac{1}{9}\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$$

22. (a) **23. (b)**

24. (b, d) : For $x = 0$, $G(0, t) = 0$, $t \geq 0$

$$g(0)=\int_0^1 f(t)\,G\,(0,t)\,dt=0$$

Also, for $x = 1$, $G(1, t) = 0$, $t < 1$

$$g(1)=\int_0^1 f(t)\,G\,(1,t)\,dt=0$$

Now, $g(x)=\int_0^x f(t)\,G\,(x,t)\,dt+\int_x^1 f(t)\,G\,(x,t)\,dt$

$$=(x-1)\int_0^x tf(t)dt+x\int_x^1(t-1)\,f(t)\,dt$$

$$\therefore\ g''(x)=xf(x)+\{0-(x-1)f(x)\}\ =f(x)$$

25. (a, c) : $\int_a^{a+nT} f(x)dx=n\int_0^T f(x)\,dx$

26. (a, d) : For $0 < x < 1$. We know that $x^2 > x^3$.
For $1 < x < 2$, we have $x^3 > x^2$.
Thus, $2^{x^2} > 2^{x^3}$ for $x \in (0, 1)$ and $2^{x^3} > 2^{x^2}$ for $x \in (1, 2)$
So, $I_1 > I_2$ and $I_4 > I_3$

27. (b, d) : Let $I=\int_{1/e}^{\cot x}\frac{dt}{t(1+t^2)}$

Put $t=\frac{1}{z}\ \Rightarrow\ dt=-\frac{1}{z^2}dz$

$$\therefore\quad I=\int_e^{\tan x}\frac{-\frac{1}{z^2}dz}{\frac{1}{z}\left(1+\frac{1}{z^2}\right)}=\int_{\tan x}^{e}\frac{zdz}{(z^2+1)}=\int_{\tan x}^{e}\frac{tdt}{(1+t^2)}$$

$$\therefore\quad \int_{1/e}^{\tan x}\frac{tdt}{(1+t^2)}+\int_{1/e}^{\cot x}\frac{dt}{t(1+t^2)}$$

$$=\int_{1/e}^{\tan x}\frac{t}{(1+t^2)}dt+\int_{\tan x}^{e}\frac{t}{(1+t^2)}dt$$

$$=\int_{1/e}^{e}\frac{tdt}{(1+t^2)}=\frac{1}{2}[\ln(1+t^2)]_{1/e}^{e}$$

$$=\frac{1}{2}\left\{\ln(1+e^2)-\ln\left(1+\frac{1}{e^2}\right)\right\}=\frac{1}{2}(\ln e^2)=1$$

Also, $\frac{2}{\pi}\int_{-1}^{1}\frac{dt}{1+t^2}=\frac{4}{\pi}\int_0^1\frac{dt}{1+t^2}=\frac{4}{\pi}\cdot\tan^{-1}1=\frac{4}{\pi}\cdot\frac{\pi}{4}=1$

28. (a, d)

29. (c) : Here, $f(x) = x^2 + 1$, $\alpha(x) = [x]$

$$\therefore\quad \int_0^3(x^2+1)d[x]+\int_0^3[x]2xdx=[3](9+1)-0$$

$$\Rightarrow\quad \int_0^3(x^2+1)d[x]=30-2\int_0^3 x[x]dx$$

$$=30-2\left\{\int_0^1 0\cdot dx+\int_1^2 xdx+\int_2^3 2xdx\right\}$$

$$=30-2\left\{0+\frac{3}{2}+5\right\}=30-3-10=17$$

30. (c) : $\int_0^6(x^2+[x])\,d\,|3-x|$

$$=\int_0^3(x^2+[x])d(3-x)+\int_3^6(x^2+[x])d(x-3)$$

$$=-\int_0^3 x^2dx+\int_3^6 x^2dx-\int_0^3[x]dx+\int_3^6[x]dx$$

$$= -\left\{\frac{x^3}{3}\right\}_0^3 + \left\{\frac{x^3}{3}\right\}_3^6 - \int_0^3 [x]dx + \int_0^3 [x+3]dx$$

$$= -9 + (72-9) - \int_0^3 [x]\,dx + \int_0^3 [x]\,dx + 3\int_0^3 1\,dx = 72 - 9 = 63$$

(31-32) : 31. (c) **32. (d)**

$$I_n = \int x^n \sqrt{a^2 - x^2}\,dx = \int x^{n-1}(x\sqrt{a^2 - x^2})dx$$

Applying integration by parts, we get

$$I_n = -\frac{1}{3}(a^2 - x^2)^{3/2} x^{n-1} + \frac{n-1}{3}\int x^{n-2}(a^2 - x^2)^{3/2}\,dx$$

$$= -\frac{1}{3}(a^2 - x^2)^{3/2} x^{n-1} + \frac{n-1}{3}\int (a^2 x^{n-2}\sqrt{a^2 - x^2} - x^n\sqrt{a^2 - x^2})dx$$

$$\Rightarrow \quad I_n = -\frac{1}{3}(a^2 - x^2)^{3/2} x^{n-1} + \frac{n-1}{3}a^2 I_{n-2} - \frac{n-1}{3}I_n$$

$$\Rightarrow \quad I_n = \frac{-(a^2 - x^2)^{3/2} x^{n-1}}{n+2} + \left(\frac{n-1}{n+2}\right)a^2 I_{n-2}$$

$$\therefore \quad A = n+2 \text{ and } B = \frac{n-1}{n+2}$$

33. (a) : If $I_n = \int_0^a x^n \sqrt{a^2 - x^2}\,dx \Rightarrow I_n = \left(\frac{n-1}{n+2}\right)a^2 I_{n-2}$

$$\therefore \quad I_4 = \frac{3}{6}a^2 I_2,\ I_2 = \frac{1}{4}a^2 I_0 \text{ and } I_0 = \frac{\pi a^2}{4}$$

$$\Rightarrow \quad I_4 = \frac{3}{6} \times \frac{1}{4} \times \frac{\pi}{4} a^6 = \frac{\pi a^6}{32}$$

34. A → s; B → p; C → r; D → q

(A) $\int \sqrt{\text{cosec}\,x + 1}\,dx = \int \frac{\sqrt{1+\sin x}}{\sqrt{\sin x}}\,dx$

$$= \int \frac{\cos x}{\sqrt{\sin x(1-\sin x)}}dx = \int \frac{dt}{\sqrt{t(1-t)}}$$

$$= \sin^{-1}(2\sin x - 1) + c$$

(B) $\int \frac{\sin x}{\sqrt{3 - \cos 2x}}\,dx = \frac{1}{\sqrt{2}}\int \frac{\sin x}{\sqrt{2 - \cos^2 x}}\,dx$

$$= \frac{-1}{\sqrt{2}}\int \frac{dt}{\sqrt{2 - t^2}} = \frac{1}{\sqrt{2}}\cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right) + c$$

(C) $\int \frac{\cos x}{\sqrt{\cos 2x}}\,dx = \frac{1}{\sqrt{2}}\int \frac{\sin 2x}{\sqrt{1 - \cos 2x}\sqrt{\cos 2x}}\,dx$

$$= -\frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{1 - t^2}}\,dt = -\frac{1}{\sqrt{2}}\sin^{-1}(\sqrt{\cos 2x}) + c$$

(D) $\int \frac{\cos 2x}{\sqrt{\cos^2 x + \sin^4 x}}\,dx = \int \frac{2\cos 2x}{\sqrt{4 - \sin^2 2x}}\,dx$

$$= \int \frac{dt}{\sqrt{t^2 + 2}} = \sin^{-1}\left(\frac{\sin 2x}{2}\right) + c$$

35. A → q ; B → p ; C → r ; D → s

(A) $\int \frac{\sec x}{(\sec x + \tan x)^2}\,dx = \int \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)^3}\,dx$

$$= \int \frac{dt}{t^3} = -\frac{1}{2}(\sec x + \tan x)^{-2} + c = \frac{-\cos^2 x}{2(1+\sin x)^2} + c$$

(B) $\int \frac{\cos x}{(\sin x - 1)(\sin x - 2)}\,dx = \int \frac{dt}{(t-1)(t-2)}$

$$= \int \left(\frac{1}{t-2} - \frac{1}{t-1}\right)dt = \log\left|\frac{t-2}{t-1}\right| + c = \log\left|\frac{\sin x - 2}{\sin x - 1}\right| + c$$

(C) $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = 2\int t\sec^2 t\,dt$

$= 2[t\tan t - \log|\sec t|] + c$

$= 2x\tan^{-1}x - \log(1 + x^2) + c$

(D) $\int (\sqrt{\tan x} + \sqrt{\cot x})dx = \int \left(y + \frac{1}{y}\right)\frac{2y}{1+y^4}\,dy$

[By putting $\tan x = y^2 \Rightarrow dx = 2ydy/(1 + y^4)$]

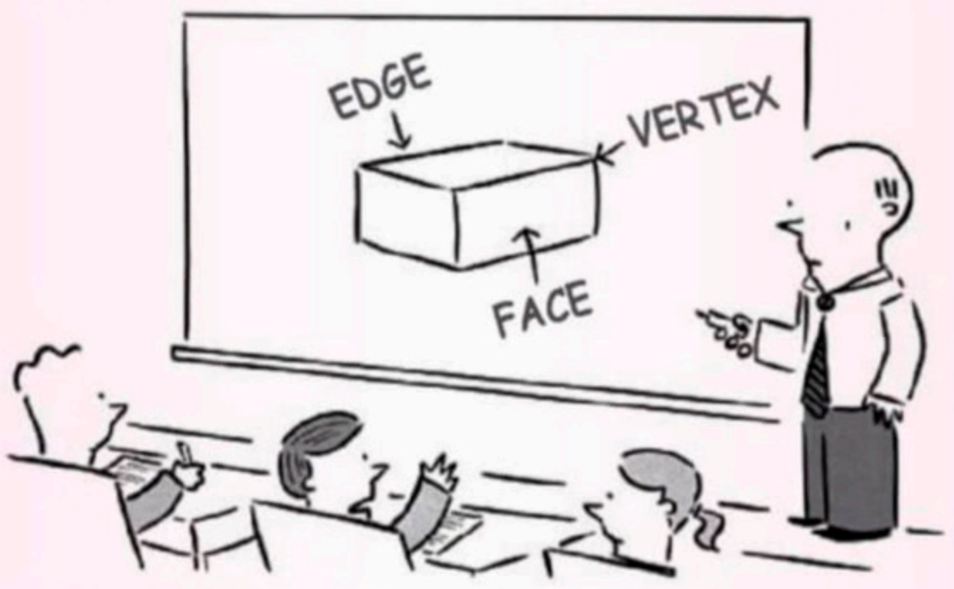
$$= 2\int \frac{y^2 + 1}{1 + y^4}\,dy = 2\int \frac{1 + 1/y^2}{y^2 + 1/y^2}\,dy$$

$$= 2\int \frac{1 + 1/y^2}{(y - 1/y)^2 + 2}\,dy = 2\int \frac{du}{u^2 + 2}$$

$$\left[\text{By putting } u = y - 1/y \Rightarrow du = \left(1 + \frac{1}{y^2}\right)dy\right]$$

$$= 2\frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}} + c = \sqrt{2}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + c$$

COMIC CAPSULE



"That's fine, but you haven't told us the most important part - what's in it?!"

CBSE warm-up!

CLASS-XII

Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Series 9 Linear Programming and Probability

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

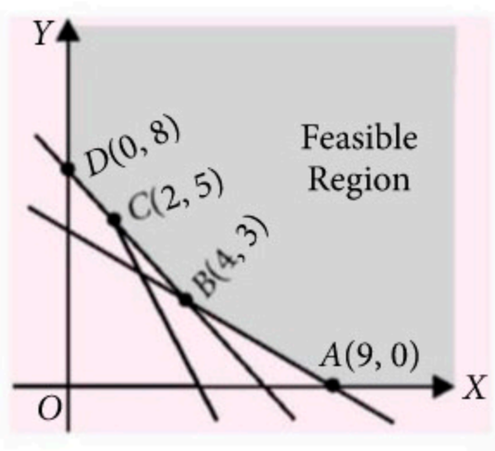
(i) All the questions are compulsory.
(ii) This question paper contains 36 questions.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION - A

Q.1-Q.10 are multiple choice type questions. Select the correct option.

1. In solving the LPP : "minimize $f = 6x + 10y$ subject to constraints $x \geq 6, y \geq 2, 2x + y \geq 10, x \geq 0, y \geq 0$" redundant constraints are
(a) $x \geq 6, y \geq 2$
(b) $2x + y \geq 10, x \geq 0, y \geq 0$
(c) $x \geq 6$ (d) none of these

2. Feasible region for an LPP is shown shaded in the following figure.



Minimum of $Z = 4x + 3y$ occurs at the point
(a) (0, 8)
(b) (2, 5)
(c) (4, 3)
(d) (9, 0)

3. Which of the following statements is correct?
(a) Every LPP has at least one optimal solution.
(b) Every LPP has a unique optimal solution.
(c) If an LPP has two optimal solutions, then it has infinitely many solutions.
(d) None of these

4. Minimize $z = \sum_{j=1}^{n}\sum_{i=1}^{m} c_{ij}\, x_{ij}$, subject to

$\sum_{j=1}^{n} x_{ij} = a_i, i=1,2,...,m$ and $\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1, 2, ..., n$

is an LPP with number of constraints
(a) $m + n$ (b) $m - n$ (c) mn (d) $\frac{m}{n}$

5. The optimal value of the objective function is attained at the points
(a) on X-axis (b) on Y-axis
(c) which are corner points of the feasible region
(d) none of these

6. If A and B are 2 events such that $P(A) > 0$ and $P(B) \neq 1$, then $P(\overline{A} \mid \overline{B}) =$

(a) $1 - P(A|B)$ (b) $1 - P(A \mid \overline{B})$

(c) $\dfrac{1 - P(A \cup B)}{P(B)}$ (d) $\dfrac{P(\overline{A})}{P(B)}$

7. If three events of a sample space are E, F and G, then $P(E \cap F \cap G)$ is equal to

(a) $P(E)\ P(F|E)\ P(G|(E \cap F))$

(b) $P(E)\ P(F|E)\ P(G|EF))$

(c) Both (a) and (b) (d) None of these

8. Two events A and B will be independent, if

(a) A and B are mutually exclusive

(b) $P(A' \cap B') = [1 - P(A)]\ [1 - P(B)]$

(c) $P(A) = P(B)$ (d) $P(A) + P(B) = 1$

9. The probability distribution of a discrete random variable X is given below :

X	0	1	2	3
$P(X)$	$\frac{4}{k}$	$\frac{6}{k}$	$\frac{10}{k}$	$\frac{12}{k}$

The value of k is

(a) 8 (b) 16 (c) 32 (d) 48

10. If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B \mid A) = 0.6$, then $P(A \cup B)$ is equal to

(a) 0.24 (b) 0.3 (c) 0.48 (d) 0.96

(Q.11 - Q.15) Fill in the blanks.

11. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement, then the probability that both drawn balls are black, is _____.

12. Let A and B be independent events with $P(A) = 1/4$ and $P(A \cup B) = 2P(B) - P(A)$. Then $P(B) =$ _____

OR

For the following probability distribution :

X	−4	−3	−2	−1	0
$P(X)$	0.1	0.2	0.3	0.2	0.2

$E(X)$ is equal to _____.

13. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).

Let $F = 4x + 6y$ be the objective function.

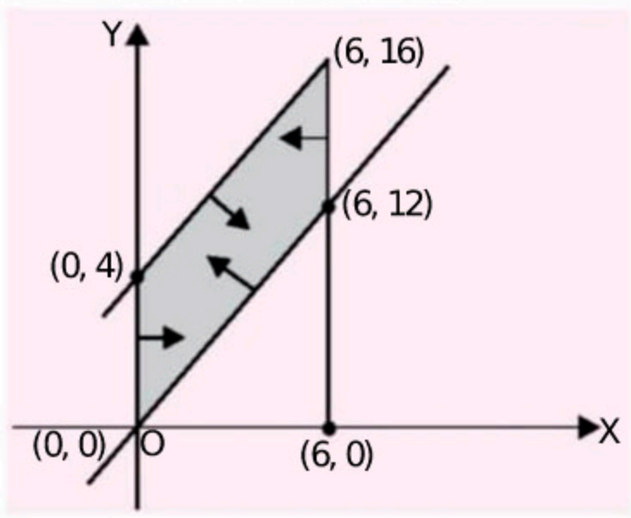
The minimum value of F occurs at _____.

14. A feasible region of a system of linear inequalities is said to be _____ , if it can be enclosed within a circle.

15. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is _____.

OR

The feasible region for an LPP is shown shaded in the figure. Let $F = 3x - 4y$ be the objective function. Maximum value of F is _____.



(Q.16-Q.20) Answer the following questions.

16. Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is neither 9 nor 11 ?

17. Suppose that five good fuses and two defective ones have been mixed up. To find the defective fuses, we test them one-by-one, at random and without replacement. What is the probability that we are lucky and find both of the defective fuses in the first two tests?

18. Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. Find the probability, that both cards drawn are queens.

19. If A and B are two independent events, then prove that the probability of occurrence of at least one of A and B is given by $1 - P(A')\ P(B')$.

20. X is taking up subjects-Mathematics, Physics and Chemistry in the examination. His probabilities of getting grade A in these subjects are 0.2, 0.3 and 0.5 respectively. Find the probability that he gets grade A in all subjects.

OR

A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

SECTION - B

21. Shamali wants to invest ₹ 50,000 in saving certificates and PPF. She wants to invest atleast ₹ 15, 000 in saving certificates and at least ₹ 20,000 in PPF. The rate of interest on saving certificates is 8% p.a. and that on PPF is 9% p.a. Formulate the above problem as LPP to determine maximum yearly income.

OR

A furniture manufacturer produces tables and bookshelves made up of wood and steel. The weekly requirement of wood and steel is given as below.

Material / Product	Wood	Steel
Table (x)	8	2
Bookshelf (y)	11	3

The weekly availability of wood and steel is 450 and 100 units respectively. Profit on a table is ₹ 1000 and that on a bookshelf is ₹ 1200. To determine the number of tables and bookshelves to be produced every week in order to maximize the total profit, formulate the problem as LPP.

22. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹ 100 and that on a bracelet is ₹ 300. Formulate an LPP for finding how many of each should be produced daily to maximize the profit. It is being given that at least one of each must be produced.

23. Two tailors, A and B, earn ₹ 300 and ₹ 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

24. If A and B are independent events, prove that $P(A \cup B) \cdot P(A' \cap B') \leq P(C)$, where C is an event defined that exactly one of A and B occurs.

25. The random variable X can take only the values 0, 1, 2. Given that $P(X = 0) = P(X = 1) = p$ and $E(X^2) = E(X)$, then find the value of p.

OR

Prove that if E and F are independent events, then the events E' and F' are also independent.

26. A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact ?

SECTION - C

27. A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white ?

28. Solve the following LPP graphically :

Minimize $Z = 3x + 5y$ subject to

$-2x + y \leq 4, x + y \geq 3, x - 2y \leq 2, x, y \geq 0$

OR

Find graphically, the maximum value of $z = 2x + 5y$, subject to constraints given below:

$2x + 4y \leq 8, 3x + y \leq 6, x + y \leq 4; x \geq 0, y \geq 0$

29. A manufacturing company makes two types of teaching aids A and B of Mathematics for class XII. Each type of A requires 9 labour hours of fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹ 80 on each piece of type A and ₹ 120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically. What is the maximum profit per week?

30. The probabilities of P, Q and R solving a problem are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. If the problem is attempted by all simultaneously, find the probability of exactly one of them solve it.

OR

Three persons A, B and C apply for a job of Manager in a Private Company. Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change does not take place, find the probability that it is due to the appointment of C.

31. Find the probability of at most two tails or at least two heads in a toss of three coins.

32. A can hit a target 4 times in 5 shots, B 3 times in 4 shots, and C 2 times in 3 shots. Calculate the probability that

(i) B, C may hit and A may not.

(ii) Any two of A, B and C will hit the target

(iii) none of them will hit the target.

SECTION - D

33. Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6 and 7. Let X denote the larger of the two numbers

obtained. Find the mean and variance of the probability distribution of X.

34. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident for them are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver or a car driver?

OR

Find the probability distribution of the number of green balls drawn when 3 balls are drawn, one by one, without replacement from a bag containing 3 green and 5 white balls.

35. Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

36. A merchant plans to sell two types of personal computers - a desktop model and a portable model that will cost ₹ 25,000 and ₹ 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit, if he does not want to invest more than ₹ 70 lakhs and his profit on the desktop model is ₹ 4500 and on the portable model is ₹ 5,000. Form it as an LPP and solve it graphically.

OR

A library has to accommodate two different types of books on a shelf. The books are 6 cm and 4 cm thick and weigh 1 kg and $1\frac{1}{2}$ kg each respectively. The shelf is 96 cm long and at most can support a weight of 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books? Form it as an LPP and solve it graphically.

SOLUTIONS

1. (b) : When $x \geq 6$ and $y \geq 2$, then $2x + y \geq 14$

Hence, $x \geq 0$, $y \geq 0$ and $2x + y \geq 10$ are automatically satisfied by every point of the region

$\{(x, y) : x \geq 6\} \cap \{(x, y) : y \geq 2\}$

2. (b) :

Corner Points	Value of $Z = 4x + 3y$	
(0, 8)	$4 \times 0 + 3 \times 8 = 24$	
(2, 5)	$4 \times 2 + 3 \times 5 = 23$	← minimum
(4, 3)	$4 \times 4 + 3 \times 3 = 25$	
(9, 0)	$4 \times 9 + 3 \times 0 = 36$	

3. (c) **4. (a)** **5. (c)** **6. (b)**

7. (c) : If E, F and G are the three events of sample space, then $P(E \cap F \cap G) = P(E)\, P(F|E)\, P(G|(E \cap F))$

$= P(E)\, P(F|E)\, P(G|EF)$

8. (b) : $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - P(A) - P(B) + P(A)\, P(B)$

$= [1 - P(A)][1 - P(B)]$ [$\because$ A and B are independent]

9. (c) : $\frac{4}{k} + \frac{6}{k} + \frac{10}{k} + \frac{12}{k} = 1 \Rightarrow \frac{32}{k} = 1 \Rightarrow k = 32$

10. (d) : $P(A) = 0.4$, $P(B) = 0.8$, $P(B|A) = 0.6$

$P(A \cap B) = P(B|A)P(A) = 0.6 \times 0.4 = 0.24$

$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.8 - 0.24 = 0.96$

11. Let E and F denote respectively the events that first and second ball drawn are black. We have to find $P(E \cap F)$.

Now, $P(E) = P$(black ball in first draw) $= \frac{10}{15}$

$P(F|E) = \frac{9}{14}$

By multiplication rule of probability, we have

$P(E \cap F) = P(E)\, .\, P(F|E) = \frac{10}{15} \times \frac{9}{14} = \frac{3}{7}$

12. We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\, P(B)$ [$\because$ Since A and B are independent]

$\therefore$ $1/4 + P(B) - (1/4)\, P(B) = 2P(B) - 1/4$ [$\because$ $P(A) = \frac{1}{4}$]

$\Rightarrow$ $P(B) = 2/5$.

OR

$E(X) = \Sigma XP(X) = (-4) \times 0.1 + (-3) \times 0.2 + (-2) \times 0.3 + (-1) \times 0.2 + 0 \times 0.2$

$= -0.4 - 0.6 - 0.6 - 0.2 = -1.8$

13.

Corner Points	Value of $F = 4x + 6y$	
(0, 2)	$4 \times 0 + 6 \times 2 = 12$	} ← minimum
(3, 0)	$4 \times 3 + 6 \times 0 = 12$	

(6, 0)	$4 \times 6 + 6 \times 0 = 24$
(6, 8)	$4 \times 6 + 6 \times 8 = 72$ ← maximum
(0, 5)	$4 \times 0 + 6 \times 5 = 30$

Since, the minimum value (F) = 12 occurs at two distinct corner points, it occurs at every point of the segment joining the points (0, 2) and (3, 0).

14. A feasible region of a system of linear inequalities is said to be bounded, if it can be enclosed within a circle.

15. We must have value of Z at (3, 0) = value of Z at (1, 1)

$$\Rightarrow \ 3p + 0 \cdot q = 1 \cdot p + 1 \cdot q \Rightarrow 3p = p + q \Rightarrow p = \frac{1}{2}q$$

OR

Construct the following table of values of the objective function F :

Corner Points	**Value of $F = 3x - 4y$**	
(0, 0)	$3 \times 0 - 4 \times 0 = 0$	← maximum
(6, 12)	$3 \times 6 - 4 \times 12 = -30$	
(6, 16)	$3 \times 6 - 4 \times 16 = -46$	← minimum
(0, 4)	$3 \times 0 - 4 \times 4 = -16$	

Hence, maximum $F = 0$.

16. If two dice are thrown, then total number of cases = 36
Number of cases of total of 9 or 11
$= \{(3, 6), (4, 5), (6, 3), (5, 4), (6, 5), (5, 6)\}$

$$P(\text{total 9 or 11}) = \frac{6}{36} = \frac{1}{6}$$

$$\therefore \quad P(\text{neither total of 9 nor 11}) = 1 - P(\text{total 9 or 11}) = 1 - \frac{1}{6} = \frac{5}{6}$$

17. Let D_1, D_2 be the events that we find a defective fuse in the first and second test respectively.
Required probability $= P(D_1 \cap D_2)$

$$= P(D_1)P(D_2 \mid D_1) = \frac{2}{7} \cdot \frac{1}{6} = \frac{1}{21}$$

18. Total number of cards = 52
Number of queens = 4

$$\text{Hence, required probability} = \frac{4}{52} \times \frac{4}{52} = \frac{1}{13} \times \frac{1}{13}$$

19. P(atleast one of A and B)
$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= P(A) + P(B) - P(A)\,P(B) \quad [\because A, B \text{ are independent}]$
$= P(A) + P(B)\,[1 - P(A)]$
$= 1 - P(A') + P(B)\,P(A') = 1 - P(A')\,[1 - P(B)]$
$= 1 - P(A')\,P(B')$

20. Let M, P' and C denote the events that X gets grade A in Mathematics, Physics and Chemistry respectively.
Given, $P(M) = 0.2$, $P(P') = 0.3$ and $P(C) = 0.5$

$$\therefore \quad P(\text{grade } A \text{ in all subjects}) = P(M \cap P' \cap C) = P(M) \cdot P(P') \cdot P(C) = 0.2 \times 0.3 \times 0.5 = 0.03$$

OR

Refer to answer 4, Page no. 329 of MTG CBSE Champion Mathematics, Class-12

21. Let Shamali invest ₹ x in saving certificates and ₹ y in PPF.

$\therefore \quad x + y \le 50000$, $x \ge 15000$ and $y \ge 20000$

$$\text{Total income} = \frac{8}{100}x + \frac{9}{100}y$$

$\therefore$ Given problem can be formulated as
Maximize $Z = 0.08\,x + 0.09y$
Subject to, $x + y \le 50000$, $x \ge 15000$, $y \ge 20000$.

OR

Let x and y units respectively of tables and bookshelves are produced.
$\therefore$ Given problem can be formulated as
Maximize $Z = 1000x + 1200y$
Subject to, $8x + 11y \le 450$, $2x + 3y \le 100$, $x \ge 0$, $y \ge 0$

22. Maximize $Z = 100x + 300y$
Subject to, $x + y \le 24$, $2x + y \le 32$, $x - 1 \ge 0$, $y - 1 \ge 0$

23. Minimize $Z = 300x + 400y$
Subject to, $3x + 5y \ge 30$, $x + y \ge 8$, $x \ge 0$, $y \ge 0$.

24. L.H.S. $= P(A \cup B) \cdot P(A' \cap B')$
$= P(A \cup B) \cdot P(A') \cdot P(B') \le (P(A) + P(B))\,P(A') \cdot P(B')$
$= P(A) \cdot P(A') \cdot P(B') + P(B) \cdot P(A') \cdot P(B')$
$\le P(A)P(B') + P(B)\,P(A')$ [since $P(A') \le 1, P(B') \le 1$]
$= P(A \cap B') + P(A' \cap B) = P(C) = \text{R.H.S.}$

25. Clearly, $P(X = 0) + P(X = 1) + P(X = 2) = 1$
$\Rightarrow \ p + p + P(X = 2) = 1$
$\Rightarrow \ P(X = 2) = 1 - 2p$.
So, the probability distribution of X is as given below :

x_i	0	1	2
p_i	p	p	$1 - 2p$

$\therefore \quad E(X) = (0 \times p) + (1 \times p) + 2\,(1 - 2p) = 2 - 3p$
and $E(X^2) = 0^2 \times p + 1^2 \times p + 2^2(1 - 2p) = 4 - 7p$

$$\text{Now, } E(X^2) = E(X) \Rightarrow 4 - 7p = 2 - 3p \Rightarrow p = \frac{1}{2}$$

OR

Refer to answer 11, page no. 330 of MTG CBSE Champion Mathematics, Class-12.

26. Let E = the event of A speaking the truth
and F = the event of B speaking the truth

$$\text{Then, } P(E) = \frac{60}{100} = \frac{3}{5} \quad \text{and} \quad P(F) = \frac{90}{100} = \frac{9}{10}$$

Required probability $= P(E\bar{F}$ or $\bar{E}F) = P(E\bar{F}) + P(\bar{E}F)$

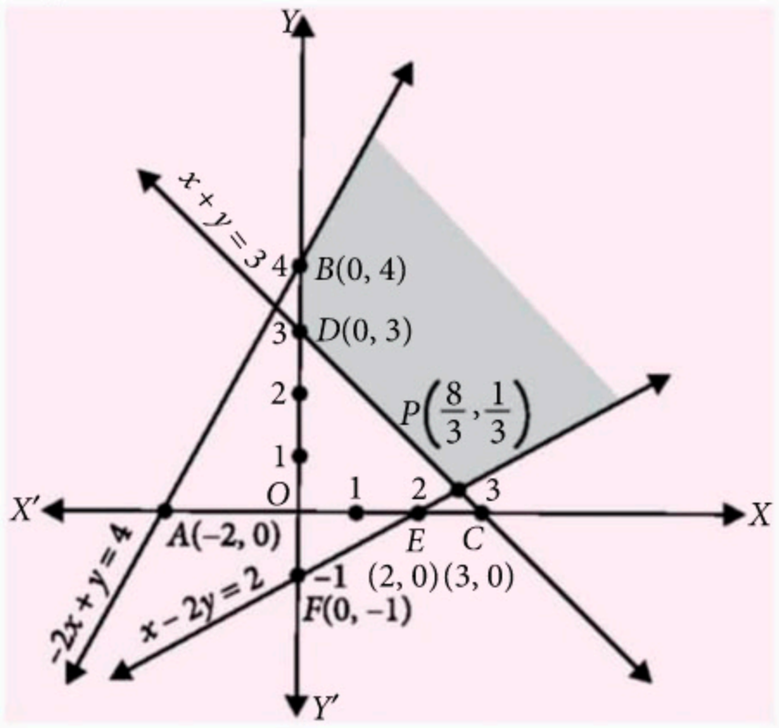
$= P(E)\cdot P(\bar{F}) + P(\bar{E})\cdot P(F)$

$= \frac{3}{5}\left(1-\frac{9}{10}\right)+\left(1-\frac{3}{5}\right)\cdot\frac{9}{10} = \frac{21}{50} = \frac{42}{100}$

$\therefore$ A and B are likely to contradict each other in 42% cases.

27. *Refer to answer 25, Page no. 333 of MTG CBSE Champion Mathematics, Class-12*

28. First we draw the lines $-2x + y = 4$, $x + y = 3$ and $x - 2y = 2$



The feasible region has been shown shaded which is unbounded. The vertices of the feasible region are :

$P\left(\frac{8}{3},\frac{1}{3}\right)$, $D\ (0,3)$ and $B\ (0,4)$.

Given, $Z = 3x + 5y$

At $P\left(\frac{8}{3},\frac{1}{3}\right)$, $Z = 3\times\frac{8}{3}+5\times\frac{1}{3} = \frac{29}{3}$ (minimum)

At $D(0, 3)$, $Z = 0 + 5 \times 3 = 15$

At $B(0, 4)$, $Z = 0 + 5 \times 4 = 20$

$\therefore$ Minimum value of $Z = \frac{29}{3}$ at $P\left(\frac{8}{3},\frac{1}{3}\right)$ *i.e.*, when $x = \frac{8}{3}$ and $y = \frac{1}{3}$

OR

Refer to answer 3, Page no. 307-308 of MTG CBSE Champion Mathematics, Class-12

29. *Refer to answer 17, Page no. 314 of MTG CBSE Champion Mathematics, Class-12*

30. Let A, B, C denote the events that the problem is solved by P, Q and R respectively.

Given $P(A) = \frac{1}{2} \Rightarrow P(A') = 1 - \frac{1}{2} = \frac{1}{2}$

$P(B) = \frac{1}{3} \Rightarrow P(B') = 1 - \frac{1}{3} = \frac{2}{3}$

$P(C) = \frac{1}{4} \Rightarrow P(C') = 1 - \frac{1}{4} = \frac{3}{4}$

Required probability

$= P[(A \cap B' \cap C') \cup (A' \cap B \cap C') \cup (A' \cap B' \cap C)]$

$= P[(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)]$

$= P(A)\,P(B')\,P(C') + P(A')\,P(B)\,P(C') + P(A')\,P(B')\,P(C)$

[$\because$ A, B, C are independent events]

$= \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} + \frac{1}{2}\times\frac{3}{4}\times\frac{1}{2} + \frac{1}{2}\times\frac{2}{3}\times\frac{1}{4} = \frac{11}{24}$

OR

Refer to answer 24, Page no. 332 of MTG CBSE Champion Mathematics, Class-12

31. On tossing three coins, sample space

$S = [HHH, HHT, HTH, THH, TTH, THT, HTT, TTT]$

$\Rightarrow$ $n(S) = 8$

Let A be the event of getting at most two tails, then

$A = [HHH, HHT, HTH, THH, TTH, THT, HTT]$

$\Rightarrow$ $n(A) = 7$ $\therefore$ $P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$

Let B be the event of getting at least two heads.

Then, $B = [HHT, HTH, THH, HHH] \Rightarrow n(B) = 4$

Therefore, $P(B) = \frac{n(B)}{n(S)} = \frac{4}{8}$

$A \cap B = \{HHH, HHT, HTH, THH\} \Rightarrow P(A \cap B) = \frac{4}{8}$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B) = \frac{7}{8}+\frac{4}{8}-\frac{4}{8} = \frac{7}{8}$

32. Consider the following events :

E : A hits the target, F : B hits the target and

G : C hits the target

We have, $P(E) = \frac{4}{5}$, $P(F) = \frac{3}{4}$ and $P(G) = \frac{2}{3}$

(i) $P(B, C$ may hit and A may not$) = P(\bar{E} \cap F \cap G)$

$= P(\bar{E})\,P(F)\,P(G)$

[$\because$ E, F, G are independent events]

$= \left(1-\frac{4}{5}\right)\times\frac{3}{4}\times\frac{2}{3} = \frac{1}{10}$

(ii) P (Any two of A, B and C will hit the target)

$= P(E \cap F \cap \bar{G}) \cup (\bar{E} \cap F \cap G) \cup (E \cap \bar{F} \cap G))$

$= P(E \cap F \cap \bar{G}) + P(\bar{E} \cap F \cap G) + P(E \cap \bar{F} \cap G))$

$= P(\mathrm{E})\,P(\mathrm{F})\,P(\bar{G}) + P(\bar{E})\,P(\mathrm{F})\,P(\mathrm{G}) + P(E)\,P(\bar{F})\,P(G)$

$= \left(\frac{4}{5}\times\frac{3}{4}\times\frac{1}{3}\right)+\left(\frac{1}{5}\times\frac{3}{4}\times\frac{2}{3}\right)+\left(\frac{4}{5}\times\frac{1}{4}\times\frac{2}{3}\right) = \frac{13}{30}$

(iii) P(None of A, B and C will hit the target)

$$= P(\bar{E} \cap \bar{F} \cap \bar{G}) = \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{60}$$

33. *Refer to answer 63, Page no. 340 of MTG CBSE Champion Mathematics, Class-12*

34. *Refer to answer 36, Page no. 335 of MTG CBSE Champion Mathematics, Class-12*

OR

Let X denotes the total number of green balls drawn in three draws without replacement. Thus, X can assume values 0, 1, 2 and 3. Let Gi denotes the event of getting a green ball in i^{th} draw.

$P(X = 0)$ = Probability of getting no green ball

$$= P(\bar{G}_1 \cap \bar{G}_2 \cap \bar{G}_3) = P(\bar{G}_1)P(\bar{G}_2 \mid \bar{G}_1)P(\bar{G}_3 \mid (\bar{G}_1 \cap \bar{G}_2))$$

$$= \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{5}{28}$$

$P(X = 1)$ = Probability of getting one green ball

$$= P\left((G_1 \cap \bar{G}_2 \cap \bar{G}_3) \cup (\bar{G}_1 \cap G_2 \cap \bar{G}_3) \cup (\bar{G}_1 \cap \bar{G}_2 \cap G_3)\right)$$

$$= \frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} + \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} + \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{15}{28}$$

$P(X = 2)$ = Probability of getting two green balls

$$= P\left((G_1 \cap G_2 \cap \bar{G}_3) \cap (\bar{G}_1 \cap G_2 \cap G_3) \cup (G_1 \cap \bar{G}_2 \cap G_3)\right)$$

$$= \frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} + \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} + \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} = \frac{15}{56}$$

$P(X = 3)$ = Probability of getting three green balls

$$= P(G_1 \cap G_2 \cap G_3)$$

$$= P(G_1)P(G_2|G_1)P(G_3|G_1 \cap G_2) = \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} = \frac{1}{56}$$

Thus, the probability distribution of the number of green balls is given by

X	0	1	2	3
$P(X)$	5/28	15/28	15/56	1/56

35. *Refer to answer 51, Page no. 337-338 of MTG CBSE Champion Mathematics, Class-12*

36. Let the merchant stocks x desktop model computers and y portable model computers.

Hence, the mathematical formulation of the problem is

Maximize $Z = 4500x + 5000y$

subject to :

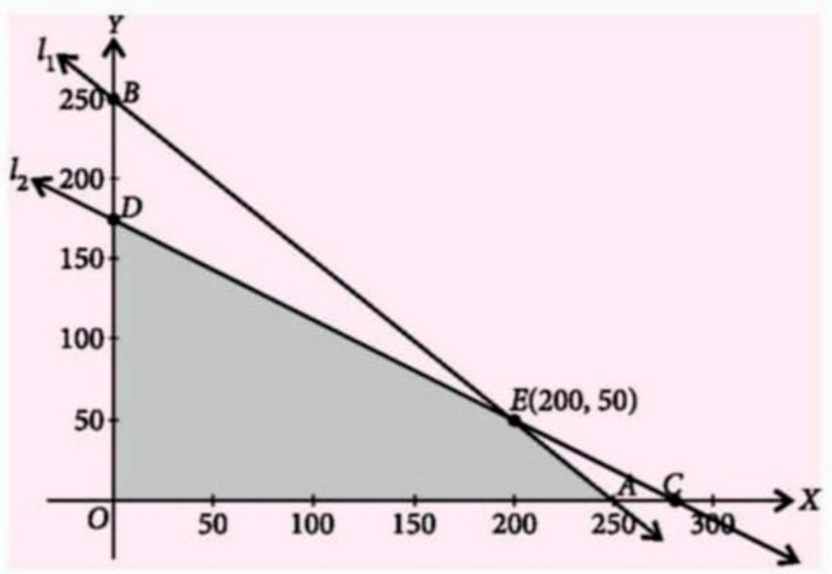
$x + y \leq 250$, $25000x + 40000y \leq 7000000$,

$5x + 8y \leq 1400$; $x, y \geq 0$

To solve LPP graphically we convert inequations into equations

$l_1 : x + y = 250$, $l_2 : 5x + 8y = 1400$ and $x = 0$, $y = 0$

These lines intersect at $E(200, 50)$



The feasible region $OAED$ is shown in the graph.

Here we observe that the feasible region is bounded.

The corner points are $O(0, 0)$, $A(250, 0)$, $E(200, 50)$ and $D(0, 175)$.

Corner Points	Value of $Z = 4500x + 5000y$	
O (0, 0)	0	
A (250, 0)	1125000	
E (200, 50)	1150000	← maximum
D (0, 175)	875000	

We find that maximum value of Z is 1150000 at $E(200, 50)$. Hence the merchant should stock 200 units of desktop model and 50 units of portable model to realise maximum profit and maximum profit is ₹ 1150000.

OR

Refer to answer 34, Page no. 318 of MTG CBSE Champion Mathematics, Class-12

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MONTHLY TEST DRIVE

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 8 : Differential Equations

Time Taken : 60 Min.

Only One Option Correct Type

1. The solution of the differential equation $\frac{dy}{dx}=\frac{x^2+y^2+1}{2xy}$ satisfying $y(1) = 1$, is

(a) a hyperbola (b) a circle

(c) $y^2 = x(1 + x) - 10$ (d) $(x - 2)^2 + (y - 3)^2 = 5$

2. If $y = y(x)$ and $\frac{2+\sin x}{y+1}\frac{dy}{dx}=-\cos x,\ y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ equals

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{1}{3}$ (d) 1

3. The sum of the squares of the perpendicular drawn from the points (0, 1) and (0, –1) to any tangent to a curve is 2. The equation of the curve, is

(a) $2y = c(x + 2)$ (b) $y = c(x \pm 1)$

(c) $y = c(x + 2)$ (d) $y = c(x \pm 2)$

4. Solution of the differential equation $\frac{\sqrt{x}\,dx+\sqrt{y}\,dy}{\sqrt{x}\,dx-\sqrt{y}\,dy}=\sqrt{\frac{y^3}{x^3}}$ is

(a) $\log\left(\frac{y}{x}\right)+\log\left|\frac{y^{3/2}}{x}\right|+\tan^{-1}\left(\frac{y}{x}\right)^{3/2}+c=0$

(b) $\frac{2}{3}\log\left(\frac{y}{x}\right)+\log\left|\frac{x^{3/2}+y^{3/2}}{x^{3/2}}\right|+\tan^{-1}\frac{y}{x}+c=0$

(c) $\frac{2}{3}\log\left(\frac{y}{x}\right)+\log\left|\frac{x+y}{x}\right|+\tan^{-1}\left(\frac{y^{3/2}}{x^{3/2}}\right)+c=0$

(d) None of these

5. The equation of a curve passing through the origin and satisfying the differential equation $\frac{dy}{dx}=(x-y)^2$, is

(a) $e^{2x}(1 - x + y) = 1 + x - y$

(b) $e^{2x}(1 + x - y) = 1 - x + y$

(c) $e^{2x}(1 - x + y) + (1 + x - y) = 0$

(d) $e^{2x}(1 + x + y) = 1 - x + y$

6. Solution of the differential equation $y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)dy = 0$, is

(a) $2\log|x| - \log|y| - \frac{1}{xy}=c$

(b) $2\log|y| - \log|x| - \frac{1}{xy}=c$

(c) $2\log|x| + \log|y| + xy = c$

(d) $2\log|y| + \log|x| - xy = c$

One or More Than One Option(s) Correct Type

7. Equation of the curve in which the subnormal is twice the square of the ordinate is given by (where c is arbitrary constant)

(a) $\log y = 2x + \log c$ (b) $y = ce^{2x}$

(c) $\log y = 2x - \log c$ (d) None of these

8. The solution of $\frac{xdx+ydy}{xdy-ydx}=\sqrt{\frac{1-x^2-y^2}{x^2+y^2}}$ is

(a) $\sqrt{x^2+y^2}=\sin\left\{\tan^{-1}\left(\frac{y}{x}\right)+c\right\}$

(b) $\sqrt{x^2+y^2}=\cos\left\{\left(\tan^{-1}\frac{y}{x}\right)+c\right\}$

(c) $\sqrt{x^2+y^2}=\left(\tan\left(\sin^{-1}\frac{y}{x}\right)+c\right)$

(d) $y=x\tan\left(c+\sin^{-1}\sqrt{x^2+y^2}\right)$

9. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis at Q. If PQ is of constant length k. Such a curve passing through $(0, k)$ is

(a) a circle with centre (0, 0)
(b) $x^2 + y^2 = k^2$
(c) $(1 - k)x^2 + y^2 = k^2$ (d) $x^2 + (1 + k^2)y^2 = k^2$

10. Solution of the differential equation $\frac{dy}{dx} = \frac{x+y+7}{2x+2y+3}$ is
(a) $6(x + y) + 11 \log (3x + 3y + 10) = 9x + c$
(b) $6(x + y) - 11 \log (3x + 3y + 10) = 9x + c$
(c) $6(x + y) - 11 \log (x + y + 10/3) = 9x + c$
(d) None of these

11. The equation of the curve satisfying the differential equation $y_2(x^2 + 1) = 2xy_1$ passing through the point (0, 1) and having slope of tangent at $x = 0$ as 3 (where y_2 and y_1 represent 2^{nd} and 1^{st} order derivative), then
(a) $y = f(x)$ is a strictly increasing function
(b) $y = f(x)$ is a non-monotonic function
(c) $y = f(x)$ has three distinct real roots
(d) $y = f(x)$ has only one negative root

12. A curve $y = f(x)$ passes through (1, 1) and tangent at $P(x, y)$ cuts the x-axis and y-axis at A and B, respectively, such that $BP : AP = 3 : 1$, then
(a) differential equation of curve is $xy' - 3y = 0$
(b) normal at(1, 1) is $x + 3y = 4$
(c) curve passes through $\left(2, \frac{1}{8}\right)$
(d) differential equation of curve is $xy' + 3y = 0$

13. The curve $y = f(x)$ is such that the area of the trapezium formed by the coordinate axes, ordinate of an arbitrary point and the tangent at this point equals half of the square of its abscissa. The equation of the curve can be
(a) $y = cx^2 \pm x$ (b) $y = cx^2 \pm 1$
(c) $y = cx \pm x^2$ (d) $y = cx^2 \pm x \pm 1$

Comprehension Type

Newton's law of cooling states that the rate at which a substance cools in moving air is proportional to the difference between the temperatures of the substance and that of the air. If the temperature of the air is 290 K. We can write it as $\frac{dT}{dt} = -k(T - 290)$, $k > 0$ constant, where T is temperature of substance.

14. The substance cools from 370 K to 330 K in 10 min, then
(a) $T = 290 + 160\ e^{-kt}$ (b) $T = 290 + 80\ e^{-kt}$
(c) $T = 290 + 40\ e^{-kt}$ (d) $T = 290 + 20\ e^{-kt}$

15. The value of k must be
(a) log 2 (b) (log 2)/40
(c) (log 2)/20 (d) (log 2)/10

Matrix Match Type

16. Match the differential equations given in Column-I with their solutions in Column-II.

Column-I		Column-II	
P.	$(x+2y^3)\frac{dy}{dx} = y$	1.	$x^2 + y^2 = x^4c^2$
Q.	$x\frac{dy}{dx} + 2y = x^2$	2.	$y = \frac{x^4 + c}{4x^2}$
R.	$(x^2+2y^2)\,dx - xydy = 0$	3.	$x = y^3 + cy$

	P	Q	R		P	Q	R
(a)	3	2	1	(b)	1	2	3
(c)	3	1	2	(d)	2	3	1

Numerical Value Type

17. Let a and b respectively be the degree and order of the differential equation of the family of circles touching the lines $y^2 - x^2 = 0$ and lying in the first and second quadrant, then $a + b =$

18. Let $y'(x) + y(x)g'(x) = g(x)g'(x)$, $y(0) = 0$, $x \in R$, where $f'(x)$ denotes $\frac{df(x)}{dx}$, and $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0$. Then the value of $y(2)$ is

19. If the solution of the differential equation $\frac{dy}{dx} - y = 1 - e^{-x}$ and $y(0) = y_0$ has a finite value, when $x \to \infty$, then the value of $\frac{2}{y_0}$ is

20. The solution of the differential equation $\frac{d^3y}{dx^3} - 8\frac{d^2y}{dx^2} = 0$ satisfying $y(0) = \frac{1}{8}$, $y_1(0) = 0$ and $y_2(0) = 1$ is $y = \frac{e^{8x} - 8x + 7}{\lambda}$, then the value of λ must be

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CONCEPT BOOSTER

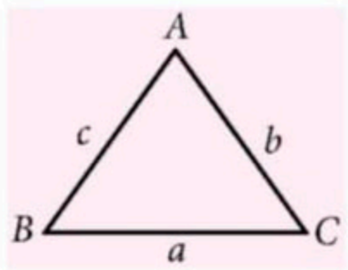
Properties of Triangles

Class XI

*ALOK KUMAR

TRIANGLE

In a triangle ABC, the angles are denoted by capital letters A, B and C and the lengths of the sides opposite to these angles are denoted by small letters a, b and c respectively. Semi-perimeter of the triangle is written as $s=\frac{a+b+c}{2}$, and its area is denoted by S or Δ.



- **Sine Rule :** $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$
- **Cosine Rule :** $\cos A=\frac{b^2+c^2-a^2}{2bc}$,

  $\cos B=\frac{a^2+c^2-b^2}{2ac}$, $\cos C=\frac{a^2+b^2-c^2}{2ab}$.
- **Projection Rule :** $a = b\cos C + c\cos B$, $b = c\cos A + a\cos C$, $c = a\cos B + b\cos A$.
- **Napier's Analogy :** $\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2}$,

  $\tan\frac{C-A}{2}=\frac{c-a}{c+a}\cot\frac{B}{2}$, $\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}$.

Trigonometric Ratios of Half Angles

- $\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$, $\sin\frac{B}{2}=\sqrt{\frac{(s-c)(s-a)}{ca}}$,

  $\sin\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{ab}}$
- $\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$, $\cos\frac{B}{2}=\sqrt{\frac{s(s-b)}{ca}}$,

  $\cos\frac{C}{2}=\sqrt{\frac{s(s-c)}{ab}}$
- $\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$, $\tan\frac{B}{2}=\sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$,

  $\tan\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$.

Area of a Triangle in Different Forms

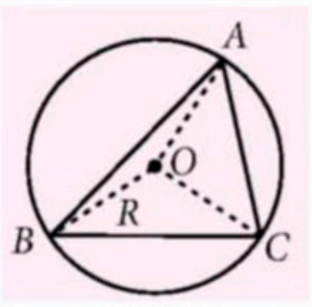
- $\Delta=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B=\frac{1}{2}ab\sin C$

  $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$, $\Delta=\frac{abc}{4R}=rs$,

  where R and r are the radii of the circumcircle and the incircle of the ΔABC.

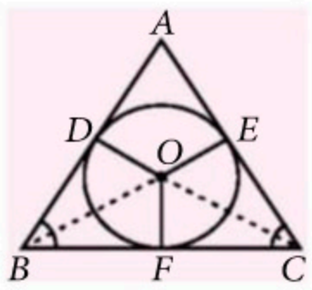
CIRCLES CONNECTED WITH TRIANGLE

Circumcircle : The circle passing through the vertices of the triangle ABC is called the circumcircle. Its radius R is called the circumradius. In ΔABC,



$R=\frac{a}{2\sin A}=\frac{b}{2\sin B}=\frac{c}{2\sin C}=\frac{abc}{4\Delta}$

Incircle : The circle touching the three sides of the triangle internally is called the incircle of the triangle. Its radius r is called the inradius of the triangle. In ΔABC,



$r=\frac{\Delta}{s}=(s-a)\tan\frac{A}{2}=(s-b)\tan\frac{B}{2}=(s-c)\tan\frac{C}{2}$

$=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Note :

From $r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$, we find that $r\le\frac{4R}{8}$

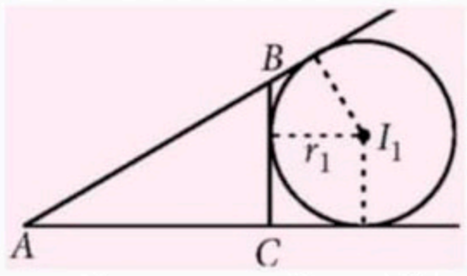
$\left[\because\ \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\le\frac{1}{8}\right]\Rightarrow 2r\le R.$

---

*Alok Kumar, a B.Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

Here equality holds for the equilateral triangle.

Escribed circles : The circle touching BC and the two sides AB and AC produced of ΔABC externally is called the escribed circle opposite to angle A. Its radius is denoted by r_1. Similarly r_2 and r_3 denote the radii of the escribed circles opposite to angles B and C respectively. r_1, r_2, r_3 are called the ex-radii of ΔABC. Here



- $r_1 = \dfrac{\Delta}{s-a} = s\tan\dfrac{A}{2} = 4R\sin\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$
- $r_2 = \dfrac{\Delta}{s-b} = s\tan\dfrac{B}{2} = 4R\sin\dfrac{B}{2}\cos\dfrac{C}{2}\cos\dfrac{A}{2}$
- $r_3 = \dfrac{\Delta}{s-c} = s\tan\dfrac{C}{2} = 4R\sin\dfrac{C}{2}\cos\dfrac{A}{2}\cos\dfrac{B}{2}$
- $r_1 + r_2 + r_3 = 4R + r$
- $r_1r_2 + r_2r_3 + r_3r_1 = s^2 = \dfrac{r_1r_2r_3}{r}$

REGULAR POLYGON

A regular polygon is a polygon which has all its sides as well as all its angles equal. If the polygon has 'n' sides, sum of its internal angles is $(n-2)\pi$ radian and each angle is $\dfrac{(n-2)\pi}{n}$ radian.

- Sum of the exterior angles of a polygon taken in one direction (clockwise or anticlockwise) remains constant and equal to 360°.
- In the regular polygon the centroid, the circumcentre, and the incentre are same.

Area of Regular Polygon : Let n is number of sides of polygon, a is length of side, R is radius of circumscribing circle and r is radius of incircle of the polygon.

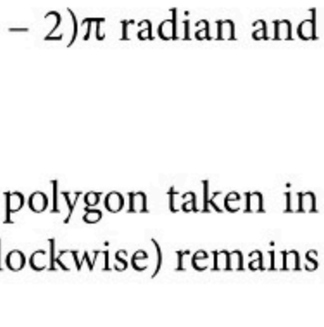
$$\text{Area of regular polygon} = \frac{na^2}{4}\cot\frac{\pi}{n} = \frac{nR^2}{2}\sin\frac{2\pi}{n}$$

$$= nr^2\tan\frac{\pi}{n}$$

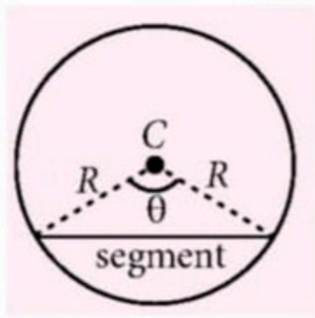
CIRCLE

Area of Sector : It is the area included between two radius and circumference.

Area of sector $= \dfrac{R^2\theta}{2}$, where θ is in radians.

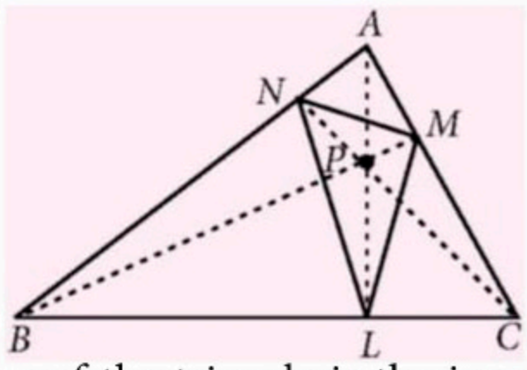


Area of Segment : It is the area between a circumference and a chord. Area of segment $= \dfrac{R^2}{2}(\theta - \sin\theta)$



PEDAL TRIANGLE

Orthocentre is a point of concurrence of altitudes from the vertices to the opposite side of the triangle. The triangle LMN which is formed by joining the feet of these three altitudes is called the pedal triangle.



- Orthocentre of the triangle is the incentre of the pedal triangle.
- If I_1, I_2 and I_3 be the centres of escribed circles which are opposite to angles A, B and C respectively and I is the centre of incircle, then triangle ABC is the pedal triangle of the triangle $I_1\ I_2\ I_3$ and I is the orthocentre of the triangle $I_1I_2I_3$.
- The centroid of the triangle lies on the line joining the circumcentre to the orthocentre and divides it in the ratio 1 : 2.
- Circle circumscribing the pedal triangle of a given triangle bisects the sides of the given triangle and also the lines joining the vertices of the given triangle to the orthocentre of the given triangle. This circle is known as nine point circle.

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- Circumcentre of the pedal triangle of a given triangle bisects the line joining the circumcentre of the triangle to the orthocentre.

CYCLIC QUADRILATERAL

A cyclic quadrilateral is a quadrilateral whose all the vertices lies on a circle.

- Sum of the opposite angles of a cyclic quadrilateral is 180°.
- In a cyclic quadrilateral sum of the products of the opposite sides is equal to the product of the diagonals. This is known as Ptolemy's theorem.
- If sum of the opposite sides of a quadrilateral is equal, then a circle can be inscribed in the quadrilateral.

SOLUTION OF TRIANGLES

The three sides a, b, c and the three angles A, B, C are called the elements of the triangle ABC. When any three of these six elements (except all the three angles) of a triangle are given, the triangle is known completely *i.e.*, the other three elements can be expressed in terms of the given elements and can be evaluated. This process is called the solution of triangles.

Problems based on finding the angles when three sides are given :

- If the data given in sine we use the formulae of trigonometric ratios of half angles in terms of sine.
- If the given data are in cosine, first of all try the formulae of trigonometric ratios of half angles in terms of cosine and see whether of logarithm of the number on R.H.S. can be determined from the given data. If proceed further, then try the cosine rule.
- If the given data are in tangent, then use the formulae of trigonometric ratios of half angles in terms of tangent.

Problem based on finding the angles when any two sides and the angles between them are given or any two sides and the difference of the angles opposite to them are given: Use the following formula whichever is needed.

- $\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2}$
- $\tan\frac{C-A}{2}=\frac{c-a}{c+a}\cot\frac{B}{2}$
- $\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}$

Problems based on finding the sides and angles when any two angles and side opposite to one of them are given:

Use the sine rule and angle sum property of triangle whichever is needed.

- When all the three angles are given : In this case unique solution of triangle is not possible. In this case only the ratio of the sides can be determined. For this only sine rule can be used
- If one side a and angles B and C are given, then $A = 180° - (B + C)$, and $b=\frac{a\sin B}{\sin A}$, $c=\frac{a\sin C}{\sin A}$.
- If two sides b and c and the angle B are given, then $\sin C=\frac{c}{b}\sin B$, $A = 180° - (B + C)$ and $a=\frac{b\sin A}{\sin B}$ gives the remaining elements.

ALGEBRAIC METHOD

By applying cosine rule, we have

$$\cos B=\frac{a^2+c^2-b^2}{2ac}$$

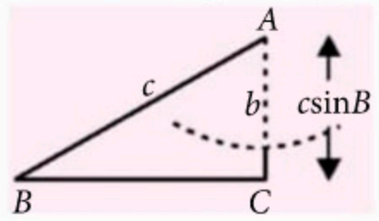
$$\Rightarrow\quad a^2-(2c\cos B)\,a+(c^2-b^2)=0$$

$$\Rightarrow\quad a=c\cos B\pm\sqrt{(c\cos B)^2-(c^2-b^2)}$$

$$\Rightarrow\quad a=c\cos B\pm\sqrt{b^2-(c\sin B)^2}$$

This equation leads to following cases :

- If $b < c\sin B$, no such triangle is possible.

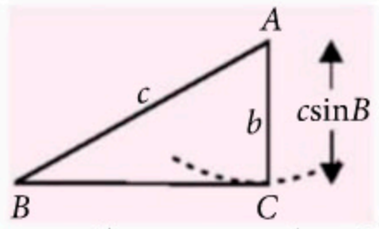


- Let $b = c\sin B$.

There are further following cases :

(i) B is an obtuse angle $\Rightarrow \cos B$ is negative. There exists no such triangle.

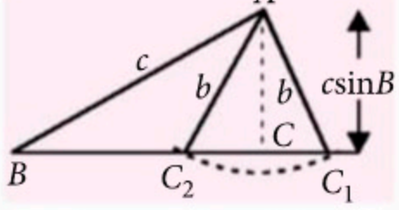
(ii) B is an acute angle $\Rightarrow \cos B$ is positive.



There exists only one such triangle.

- Let $b > c\sin B$. There are further following cases:

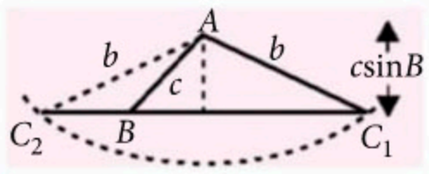
(i) B is an acute angle $\Rightarrow \cos B$ is positive.



In this case two values of a will exists if and only if $c\cos B > \sqrt{b^2-(c\sin B)^2}$ or $c > b$

$\Rightarrow$ Two such triangles are possible.

If $c < b$, only one such triangle is possible.



(ii) B is an obtuse angle

$\Rightarrow$ $\cos B$ is negative.

In this case triangle will exist if and only if

$\sqrt{b^2-(c\sin B)^2} > |c\cos B|$

$\Rightarrow$ $b > c$. So in this case only one such triangle is possible. If $b < c$ there exists no such triangle.

PROBLEMS

Single Correct Answer Type

1. If the length of arc AB, BC and CA of a circle is 3, 4 and 5 respectively. Then the area of triangle ABC is

(a) $\dfrac{9\sqrt{3}(\sqrt{3}+1)}{\pi^2}$ (b) $\dfrac{9\sqrt{3}(\sqrt{3}-1)}{\pi^2}$

(c) $\dfrac{9\sqrt{3}(\sqrt{3}-1)}{\pi}$ (d) none of these

2. The area of right angled triangle ABC in terms of r and r_1 if $\angle A = 90°$, is (where r, r_1 have their usual meanings)

(a) $r + r_1$ (b) rr_1 (c) $r - r_1$ (d) $r_1 - r$

3. The number of points inside or on the circle $x^2 + y^2 = 4$ satisfying $\tan^4 x + \cot^4 x + 1 = 3\sin^2 y$ is

(a) one (b) two (c) four (d) infinite

4. If in a ΔABC (whose circumcentre is origin), $a \le \sin A$, then for any point (x, y) inside the circumcircle of ΔABC

(a) $|xy| < 1/8$ (b) $|xy| > 1/8$

(c) $1/8 < xy < 1/2$ (d) none of these

5. If the sine of the angles of a triangle ABC satisfy the equation $c^3x^3 - c^2(a+b+c)x^2 + \lambda x + \mu = 0$ (where a, b, c are the sides of ΔABC), then triangle ABC is

(a) always right angled for any λ, μ

(b) right angled only when $\lambda = c(ab+bc+ca)$, $\mu = -abc$

(c) right angled only when $\lambda = \dfrac{c(ab+bc+ca)}{4}$, $\mu = \dfrac{-abc}{8}$

(d) never right angled

6. If in triangle ABC, line joining the circumcentre and orthocenter is parallel to side AC, then value of $\tan A \tan C$ is equal to

(a) $\sqrt{3}$ (b) 3

(c) $3\sqrt{3}$ (d) none of these

7. AA_1, BB_1 and CC_1 are the medians of triangle ABC whose centroid is G. If the points A, C_1, G and B_1 are concyclic, then

(a) $2b^2 = a^2 + c^2$ (b) $2c^2 = a^2 + b^2$

(c) $2a^2 = b^2 + c^2$ (d) none of these

8. The area of a triangle ABC, where $a = 2(\sqrt{3}+1)$ $\angle B = 45°$ and $\angle C = 60°$, is

(a) $\sqrt{3}(\sqrt{3}+1)$ sq. units (b) $2(\sqrt{3}+1)$ sq. units

(c) $2\sqrt{3}(\sqrt{3}+1)$ sq. units (d) $\sqrt{3}(2\sqrt{3}+1)$ sq. units

9. In a ΔABC, $r_1 + r_2 + r_3 - r =$

(a) R (b) $2R$ (c) $3R$ (d) $4R$

10. In a triangle ABC, $a^2b^2c^2(\sin 2A + \sin 2B + \sin 2C) =$

(a) Δ^3 (b) $8\Delta^3$ (c) $16\Delta^3$ (d) $32\Delta^3$

11. If the angles of a triangle are 30° and 45° and the included side is $(\sqrt{3}+1)$ cm, then the area (in cm^2) of the triangle is

(a) $\dfrac{1}{2}(\sqrt{3}-1)$ (b) $\dfrac{1}{2}(\sqrt{3}+1)$

(c) $\sqrt{3}+1$ (d) $2(\sqrt{3}+1)$

12. In any ΔABC, $b^2 \sin 2C + c^2 \sin 2B$ is equal to

(a) Δ (b) 2Δ (c) 3Δ (d) 4Δ

13. If p_1, p_2, p_3 are respectively the perpendiculars from the vertices of a triangle to the opposite sides, then $p_1p_2p_3$ is equal to

(a) $\dfrac{a^2b^2c^2}{R^2}$ (b) $\dfrac{a^2b^2c^2}{4R^2}$ (c) $\dfrac{4a^2b^2c^2}{R^2}$ (d) $\dfrac{a^2b^2c^2}{8R^3}$

14. If $\Delta = a^2 - (b-c)^2$, where Δ is the area of triangle ABC, then $\tan A$ is equal to

(a) $\dfrac{15}{16}$ (b) $\dfrac{8}{15}$ (c) $\dfrac{8}{17}$ (d) $\dfrac{1}{2}$

15. If $b + c = 3a$, then the value of $\cot\dfrac{B}{2}\cot\dfrac{C}{2}$ is equal to

(a) 1 (b) 2 (c) $\sqrt{3}$ (d) $\sqrt{2}$

Monthly Test Drive-8 CLASS XI ANSWER KEY

1. (b)	2. (d)	3. (b)	4. (a)
5. (c)	6. (b)	7. (a)	8. (a, b)
9. (a,b,c,d)	10. (b, d)	11. (a,b,c,d)	12. (a, b)
13. (b,d)	14. (b)	15. (d)	16. (c)
17. (4)	18. (13)	19. (1)	20. (2)

16. If in a ΔABC, $\cos B \cos C + \sin A \sin B \sin C = 1$, then $a : b : c$ is equal to

(a) $1:1:\sqrt{2}$ (b) $\sqrt{2}:1:1$

(c) $1:\sqrt{2}:1$ (d) none of these

17. In a ΔABC, $\sum \sin\frac{A}{2} = \frac{6}{5}$ and $\sum II_1 = 9$ where I_1, I_2, I_3 are ex-centres and I is incentre, then circumradius $R =$

(a) $\frac{15}{2}$ (b) $\frac{15}{4}$ (c) $\frac{15}{8}$ (d) $\frac{1}{3}$

18. $ABCD$ is a quadrilateral, $AB = a$, $BC = b$, $CD = c$, $DA = d$, is inscribed to a circle and circumscribed to another circle. Then the value $\tan^2\frac{A}{2} =$

(a) $\frac{ad}{bc}$ (b) $\frac{ab}{cd}$ (c) $\frac{bc}{ad}$ (d) $\frac{ac}{bd}$

19. If a, b, c are the sides of a triangle, then the minimum value of $\frac{2a}{b+c-a}+\frac{2b}{c+a-b}+\frac{2c}{a+b-c}$ is

(a) 3 (b) 6 (c) 8 (d) 1/8

20. If in a triangle the ex-radii r_1, r_2, r_3 are in the ratio $1 : 2 : 3$, then their sides are in the ratio

(a) $5 : 8 : 9$ (b) $1 : 2 : 3$

(c) $3 : 5 : 7$ (d) $1 : 5 : 9$

21. If P is a point on the altitude AD of the triangle ABC such that $\angle DBP = \frac{B}{3}$, then AP is equal to

(a) $2a\sin\frac{C}{3}$ (b) $2b\sin\frac{C}{3}$

(c) $2c\sin\frac{B}{3}$ (d) $2c\sin\frac{C}{3}$

22. If in a triangle ABC, $(r_2 - r_1)(r_3 - r_1) = 2r_2r_3$, then the triangle is

(a) Right angled (b) Isosceles

(c) Equilateral (d) Right angled isosceles

23. In a triangle ABC, medians AD and CE are drawn. If $AD = 5$, $\angle DAC = \frac{\pi}{8}$ and $\angle ACE = \frac{\pi}{4}$, then the area of triangle ABC is equal to

(a) $\frac{25}{9}$ (b) $\frac{25}{3}$ (c) $\frac{25}{18}$ (d) $\frac{10}{3}$

24. Let S_1 and S_2 be the areas of inscribed and circumscribed polygons of n sides respectively and S_3 is the area of regular polygon of $2n$ sides inscribed in a circle, then

(a) $2S_3 = S_1 + S_2$ (b) $S_3^2 = S_1S_2$

(c) $\frac{1}{S_3}=\frac{1}{S_1}+\frac{1}{S_2}$ (d) $\frac{2}{S_3}=\frac{1}{S_1}+\frac{1}{S_2}$

Multiple Correct Answer Type

25. If Δ is the area and $2s$ the sum of sides of a triangle, then

(a) $\Delta < \frac{s^2}{4}$ (b) $\Delta \le \frac{s^2}{3\sqrt{3}}$

(c) $\Delta > \frac{s^2}{\sqrt{3}}$ (d) $\Delta \le \frac{s^2}{\sqrt{3}}$

26. If the angles of a ΔABC satisfy, $\frac{\cos A + 2\cos C}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$, then ΔABC

(a) may be isosceles (b) must be equilateral

(c) may be a right triangle

(d) must be acute angled

27. Triangle ABC is right angled at point A, radii of described circles are r_1, r_2, r_3, given that $r_2 = 2$, $r_3 = 3$, then which of the following hold good?

(a) $a = 5$ (b) $r_1 = 6$

(c) $r = 1$ (d) None of these

28. In triangle ABC if $2a^2b^2 + 2b^2c^2 = a^4 + b^4 + c^4$, then angle B is equal to

(a) 45° (b) 135° (c) 120° (d) 60°

29. In ΔABC, which of the following statements are true?

(a) Maximum value of $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ is $\frac{1}{4}$

(b) $R \ge 2r$, where R is circumradius and r is inradius

(c) $R^2 \ge \frac{abc}{a+b+c}$

(d) ΔABC is right angled if $r + 2R = s$, where 's' is semi perimeter

Comprehension Type

Paragraph for Q. No. 30 to 32

Let ABC be any triangle and P be a point inside it such that $\angle PAB = \frac{\pi}{18}$, $\angle PBA = \frac{\pi}{9}$, $\angle PCA = \frac{\pi}{6}$, $\angle PAC = \frac{2\pi}{9}$.

Let $\angle PCB = x$

30. $x =$

(a) $\frac{\pi}{9}$ (b) $\frac{2\pi}{9}$

(c) $\frac{\pi}{3}$ (d) None of these

31. ΔABC is

(a) Equilateral (b) Isosceless

(c) Scalene (d) Right angled

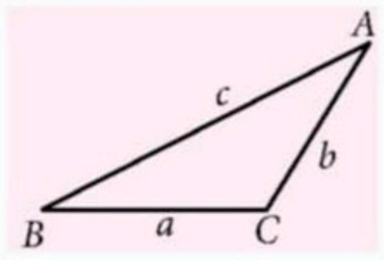
32. Which of the following is true?

(a) $BC > AC$ (b) $AC < AB$

(c) $AC > AB$ (d) $BC = AC$

Paragraph for Q. No. 33 to 35

ABC is a triangle such that $\angle C > 90°$, $\angle A = 2\angle B$ and the side lengths are integers. We wish to find the minimum perimeter.



33. A relation among a, b and c is

(a) $a^2 - 2b^2 = bc$ (b) $a^2 - b^2 = 2bc$

(c) $a^2 - b^2 = bc$ (d) $a^2 - 3b^2 = bc$

34. To seek the minimum perimeter, we must assume that $gcd\ (a, b, c) = 1$. There exist positive integers m and n such that $b = m^2$, $b + c = n^2$. This leads to $\cos B =$

(a) $\frac{2n}{m}$ (b) $\frac{n}{2m}$ (c) $\frac{n}{m}$ (d) $\frac{n}{3m}$

35. Using the range of B, we finally get the minimum perimeter =

(a) 77 (b) 83 (c) 87 (d) 93

Matrix-Match Type

36. Match the following :

Column I		Column II	
A.	If in a triangle ABC, $\sin^2 A + \sin^2 B = \sin(A + B)$, then the triangle must be	p.	right angled
B.	If in a triangle ABC, $\frac{bc}{2\cos A} = b^2 + c^2 - 2bc\ \cos A$, then the triangle must be	q.	equilateral
C.	If in a triangle ABC, $\tan\frac{A}{2} + \tan\frac{B}{2} + \tan\frac{C}{2} = \sqrt{3}$, then the triangle must be	r.	isosceles
D.	If in a triangle the sides and the altitudes are in A.P., then the triangle must be	s.	obtuse angled

Numerical Value Type

37. If the median AD of triangle ABC makes an angle $\frac{\pi}{4}$ with the side BC, then the value of $|\cot B - \cot C|$ is

38. In ΔABC if $\cos A + 2\cos B + \cos C = 2$, then the value of $\frac{2s}{b}$ (where 's' is the semi perimeter) is

39. In ΔABC if $\angle C = 90°$, then the value of $\frac{a+b}{r+R}$ is

40. The lengths of the tangents drawn from the vertices A, B, C to the incircle of ΔABC are 5, 3, 2 respectively. If the lengths of the parts of tangents within the triangle which are drawn parallel to the sides BC, CA, AB of the triangle to the incircle be α, β, γ respectively, then $[\alpha + \beta + \gamma]$ equals (where $[\cdot]$ denotes the greatest integer function)

SOLUTIONS

1. (a) : Angles subtended by the chords AB, BC and CA at centre of circle are in ratio 3 : 4 : 5 *i.e.*, 90° : 120° : 150°.

$\therefore$ $\angle B = 75°$, $\angle C = 45°$, $\angle A = 60°$

Also, circumference of circle, $2\pi R = 12$

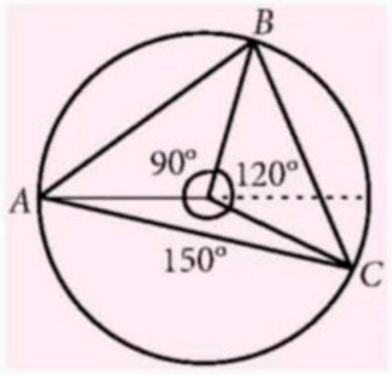
$\therefore$ Radius, $R = \frac{12}{2\pi} = \frac{6}{\pi}$

$\therefore$ Area of triangle $= 2R^2 \sin A \sin B \sin C$

$$= 2\left(\frac{6}{\pi}\right)^2 \sin 60° \sin 75° \sin 45°$$

$$= \frac{72}{\pi^2} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$= \frac{9\sqrt{3}\,(\sqrt{3}+1)}{\pi^2}$$



2. (b) : $r_1 = s \tan\frac{A}{2} = s$ and $r = (s - a) \tan\frac{A}{2} = s - a$

Also, $a^2 = b^2 + c^2 \Rightarrow r_1 + r = b + c$ and $r_1 - r = a$

Now, $(r_1 + r)^2 - (r_1 - r)^2 = (b + c)^2 - a^2 \Rightarrow 4r_1 r = 2bc$

$\Rightarrow$ $\Delta = \frac{1}{2}bc = rr_1$

3. (c) : $\tan^4 x + \cot^4 x + 1 = (\tan^2 x - \cot^2 x)^2 + 3 \geq 3$

Also, $3 \sin^2 y \leq 3 \Rightarrow \tan^2 x = \cot^2 x$, $\sin^2 y = 1$

$\Rightarrow$ $\tan x = \pm 1$, $\sin y = \pm 1$

$\Rightarrow$ $x = \pm \pi/4, \pm 3\pi/4, \ldots$

But $x^2 \leq 4 \Rightarrow -2 \leq x \leq 2 \Rightarrow x = \pm \pi/4$ only

$\sin y = \pm 1 \Rightarrow y = \pm \pi/2, \pm 3\pi/2, \ldots$

But $y^2 \leq 4 \Rightarrow y = \pm \pi/2$ only.

So, four solutions are possible.

4. (a) : $a \leq \sin A \Rightarrow \frac{a}{\sin A} \leq 1 \Rightarrow R \leq 1/2$

So for any point (x, y) inside the circumcircle, $x^2 + y^2 < 1/4 \Rightarrow |xy| < 1/8$

5. (b) : $\sin A + \sin B + \sin C$

$$= \frac{c^2(a+b+c)}{c^3} = \frac{a+b+c}{c} \quad \ldots(i)$$

But, $\sin A + \sin B + \sin C = \frac{a}{2R} + \frac{b}{2R} + \frac{c}{2R}$ ...(ii)

Comparing (i) and (ii), we get

$c = 2R \Rightarrow$ Triangle is a right angled triangle

Putting same value of c, we get $\lambda = c\,(ab + bc + ca)$, $\mu = -abc$

6. (b) : Distance of circumcentre from side AC = $R\cos B$ and distance of orthocentre from side AC

$= 2R\cos A\cos C$

$\Rightarrow R\cos B = 2R\cos A\cos C$

$\Rightarrow -\cos(A + C) = 2\cos A\cos C$

$\Rightarrow \sin A\sin C = 3\cos A\cos C \Rightarrow \tan A\tan C = 3$

7. (c) : Since A, C_1, G and B_1 are concyclic, therefore $BG \cdot BB_1 = BC_1 \cdot BA$

$$\Rightarrow \frac{2}{3}(BB_1)^2 = \frac{c}{2}\cdot c \Rightarrow \frac{2}{3}\cdot\frac{1}{4}(2a^2 + 2c^2 - b^2) = \frac{c^2}{2}$$

$\Rightarrow 2a^2 + 2c^2 - b^2 = 3c^2 \Rightarrow b^2 + c^2 = 2a^2$

8. (c) : Now, $\angle A = 180° - (45° + 60°) = 75°$

$$\frac{b}{\sin 45°} = \frac{a}{\sin 75°} \Rightarrow b = \frac{a/\sqrt{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{2a}{\sqrt{3}+1} \Rightarrow ab = \frac{2a^2}{\sqrt{3}+1}$$

$$\therefore \quad \Delta = \frac{1}{2}ab\sin C = \frac{1}{2}\frac{2a^2}{\sqrt{3}+1}.\sin 60° = 4(\sqrt{3}+1).\frac{\sqrt{3}}{2}$$

$$= 2\sqrt{3}\,(\sqrt{3}+1)\,\text{sq. units.}$$

9. (d) : $r_1 + r_2 + r_3 - r = \frac{\Delta}{s-a} + \frac{\Delta}{s-b} + \frac{\Delta}{s-c} - \frac{\Delta}{s}$

$$= \Delta\left[\frac{2s-a-b}{(s-a)(s-b)}\right] + \frac{\Delta c}{s(s-c)}$$

$$= \Delta c\left[\frac{1}{(s-a)(s-b)} + \frac{1}{s(s-c)}\right]$$

$$= \Delta c\left[\frac{s^2 - sc + s^2 - sa - sb + ab}{s(s-a)(s-b)(s-c)}\right]$$

$$= \Delta c\frac{[2s^2 - s(a+b+c) + ab]}{\Delta^2} = \frac{\Delta abc}{\Delta^2} = \frac{abc}{\Delta} = 4R$$

10. (d) : $a^2b^2c^2(\sin 2A + \sin 2B + \sin 2C)$

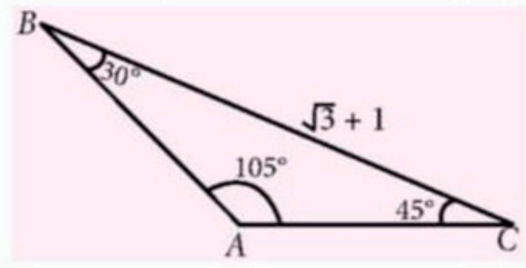
$= a^2b^2c^2\,(2\sin(A + B)\cos(A - B) + 2\sin C\cos C)$

$= a^2b^2c^2\ 2\sin C[\cos(A - B) - \cos(A + B)]$

$= a^2b^2c^2\ [4\sin A\sin B\sin C]$

$$= \frac{4a^2b^2c^2abc}{8R^3} = 32\left(\frac{abc}{4R}\right)^3 = 32\Delta^3$$

11. (b) : We have, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



$$\Rightarrow \frac{\sin 105°}{\sqrt{3}+1} = \frac{\sin 30°}{b} = \frac{\sin 45°}{c}$$

$$\Rightarrow b = \frac{(\sqrt{3}+1)}{2\sin 105°},\ c = \frac{\sqrt{3}+1}{\sqrt{2}\sin 105°}$$

$$\therefore \quad \Delta = \frac{1}{2}bc\sin A = \frac{1}{2}\frac{(\sqrt{3}+1)^2}{2\sqrt{2}\sin(60°+45°)}$$

$$= \frac{(\sqrt{3}+1)^2}{4\sqrt{2}\left[\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2}\right]} = \frac{(\sqrt{3}+1)^2}{2(\sqrt{3}+1)} = \frac{1}{2}(\sqrt{3}+1)$$

12. (d) : $b^2\sin 2C + c^2\sin 2B$

$= b^2\cdot 2\sin C\cos C + c^2\cdot 2\sin B\cos B$

$= 2(b\sin C)\,(b\cos C) + 2(c\sin B)(c\cos B)$

$= 2c\sin B\,(b\cos C + c\cos B) = 2ac\sin B = 4\Delta$

13. (d) : We have, $\Delta = \frac{1}{2}ap_1 = \frac{1}{2}bp_2 = \frac{1}{2}cp_3$

$$\therefore \quad p_1 = \frac{2\Delta}{a};\ p_2 = \frac{2\Delta}{b};\ p_3 = \frac{2\Delta}{c}$$

$$\therefore \quad p_1p_2p_3 = \frac{8\Delta^3}{abc} = \frac{8}{abc}\left(\frac{abc}{4R}\right)^3 = \frac{a^2b^2c^2}{8R^3}$$

14. (b) : $\Delta = a^2 - (b - c)^2 = 2bc - (b^2 + c^2 - a^2)$

$= 2bc(1 - \cos A) = 2bc\cdot 2\sin^2\frac{A}{2}$

Also, $\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}(bc)\,2\sin\frac{A}{2}\cos\frac{A}{2}$

$$= bc\sin\frac{A}{2}\cos\frac{A}{2}$$

$$\therefore \quad \tan\frac{A}{2} = \frac{1}{4} \qquad \therefore \quad \tan A = \frac{2\tan\frac{A}{2}}{1 - \tan^2\frac{A}{2}} = \frac{8}{15}$$

15. (b) : $\cot\frac{B}{2}\cot\frac{C}{2} = \frac{s}{s-a} = \frac{2s}{2s-2a}$

$$= \frac{a+b+c}{b+c-a} = \frac{4a}{2a} = 2$$

16. (b) : $\cos B\cos C + \sin A\sin B\sin C = 1$

$\Rightarrow \cos(B-C) = 1 + \sin B\sin C(1-\sin A)$ ...(i)

L.H.S. of (i) ≤ 1 and R.H.S. ≥ 1

$\therefore$ (i) is true iff $B - C = 0$ and $1 - \sin A = 0$

$\Rightarrow B = C$ and $A = \pi/2 \ \therefore \ B = C = \dfrac{\pi}{4}$

Thus, $a : b : c = \sin A : \sin B : \sin C = \sqrt{2} : 1 : 1$

17. (c) : $\sum I I_1 = \sum 4R\sin\dfrac{A}{2} \Rightarrow 9 = 4R \times \dfrac{6}{5}$

$\Rightarrow \quad R = \dfrac{45}{24} = \dfrac{15}{8}$

18. (c) : $\cos A = \dfrac{ad-bc}{ad+bc} = \dfrac{1-\frac{bc}{ad}}{1+\frac{bc}{ad}} = \dfrac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}$

$\therefore \ \tan^2\dfrac{A}{2} = \dfrac{bc}{ad}$

19. (b) : $\dfrac{a}{s-a} + \dfrac{b}{s-b} + \dfrac{c}{s-c}$

$= -3 + \left(\dfrac{s}{s-a} + \dfrac{s}{s-b} + \dfrac{s}{s-c}\right) \ge -3 + 9 = 6$

$\left[\because \ (x_1+x_2+x_3)\left(\dfrac{1}{x_1}+\dfrac{1}{x_2}+\dfrac{1}{x_3}\right) \ge 9\right]$

20. (a) : $r_1 : r_2 : r_3 = 1 : 2 : 3 \Rightarrow \dfrac{1}{r} = \dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3} = \dfrac{11}{6}$

$\therefore \quad a : b : c = \sqrt{(r_1 - r)(r_2 + r_3)} : \sqrt{(r_2 - r)(r_1 + r_3)} : \sqrt{(r_3 - r)(r_1 + r_2)}$

$= 5 : 8 : 9$

21. (c) : $\angle DBP = \dfrac{B}{3} \quad \therefore \quad \angle ABP = \dfrac{2B}{3}$

Now, $\dfrac{AP}{\sin\frac{2B}{3}} = \dfrac{c}{\sin\left(90^\circ + \frac{B}{3}\right)} \Rightarrow AP = 2c\sin\dfrac{B}{3}$

22. (a) : $\left(\dfrac{\Delta}{s-b} - \dfrac{\Delta}{s-a}\right)\left(\dfrac{\Delta}{s-c} - \dfrac{\Delta}{s-a}\right) = 2\left(\dfrac{\Delta}{s-b}\right)\left(\dfrac{\Delta}{s-c}\right)$

$\Rightarrow \quad (b-a)(c-a) = 2(s-a)^2$

$\Rightarrow \quad 2(b-a)(c-a) = (b+c-a)^2 \ \Rightarrow \ b^2 + c^2 = a^2$

23. (b) : $AG = \dfrac{2}{3}AD = \dfrac{10}{3}$

Now, $\dfrac{GC}{\sin\frac{\pi}{8}} = \dfrac{AG}{\sin\frac{\pi}{4}} \Rightarrow GC = \dfrac{10}{3} \times \dfrac{\sin\frac{\pi}{8}}{\sin\frac{\pi}{4}}$

$\therefore \quad$ Area of $\Delta ABC = 3$ Area of ΔAGC

$= 3\left(\dfrac{1}{2} \times \dfrac{10}{3} \times \left(\dfrac{10}{3} \times \dfrac{\sin\frac{\pi}{8}}{\sin\frac{\pi}{4}}\right)\right) \times \sin\left(\dfrac{\pi}{2} + \dfrac{\pi}{8}\right) = \dfrac{25}{3}$

24. (b) : $S_1 = n \times \dfrac{1}{2} \times r^2 \times \sin\dfrac{2\pi}{n}; \ S_2 = n.r^2\tan\dfrac{\pi}{n};$

$S_3 = \dfrac{2n}{2} \times r^2\sin\dfrac{\pi}{n}; \ S_3^2 = n^2r^4\sin^2\dfrac{\pi}{n}$

Now, $S_1S_2 = n^2r^4\dfrac{1}{2} \times 2\sin\dfrac{\pi}{n}\cos\dfrac{\pi}{n}.\dfrac{\sin\frac{\pi}{n}}{\cos\frac{\pi}{n}}$

$= n^2r^4\sin^2\dfrac{\pi}{n} = S_3^2$

25. (a, b) : $\because \ \dfrac{s+s-a+s-b+s-c}{4}$

$\ge \sqrt[4]{s(s-a)(s-b)(s-c)}$

$\Rightarrow \ \dfrac{s}{2} \ge \Delta^{1/2} \Rightarrow \dfrac{s^2}{4} \ge \Delta \Rightarrow \Delta \le \dfrac{s^2}{4}$

Now, $\dfrac{s-a+s-b+s-c}{3} \ge \sqrt[3]{(s-a)(s-b)(s-c)}$

$\Rightarrow \ \dfrac{s^3}{27} \ge \dfrac{(s-a)(s-b)(s-c)s}{s} \Rightarrow \Delta^2 \le \dfrac{s^4}{27} \Rightarrow \Delta \le \dfrac{s^2}{3\sqrt{3}}$

26. (a, c) : We have, $\sin C\cos A + \sin 2C$

$= \sin B\cos A + \sin 2B$

$\Rightarrow \ (\sin C - \sin B)\cos A = \sin 2B - \sin 2C$

$\Rightarrow \ \left(-2\cos\left(\dfrac{B+C}{2}\right)\sin\left(\dfrac{B-C}{2}\right)\right)\cos A$

$= 2\cos(B+C)\sin(B-C)$

$\Rightarrow \ -2\cos A\sin\left(\dfrac{B-C}{2}\right)\cos\left(\dfrac{B+C}{2}\right)$

$= -2\cos A \cdot 2\sin\left(\dfrac{B-C}{2}\right)\cos\left(\dfrac{B-C}{2}\right)$

$\Rightarrow \ \cos A = 0 \ \text{ or } \ \sin\dfrac{B-C}{2} = 0 \Rightarrow A = \dfrac{\pi}{2} \text{ or } B = C$

27. (a, b, c) : Since, $r_1 + r_2 + r_3 - r = 4R$

28. (a, b) : $a^4 + b^4 + c^4 - 2a^2b^2 - 2b^2c^2 = 0$

$\Rightarrow \ (a^2 - b^2 + c^2)^2 = 2c^2a^2 \Rightarrow \left(\dfrac{a^2-b^2+c^2}{2ac}\right)^2 = \pm\dfrac{1}{2}$

$\Rightarrow \ \cos^2 B = \dfrac{1}{2} \Rightarrow \cos B = \pm\dfrac{1}{\sqrt{2}}$

$\therefore \quad B = 45^\circ \text{ or } 135^\circ$

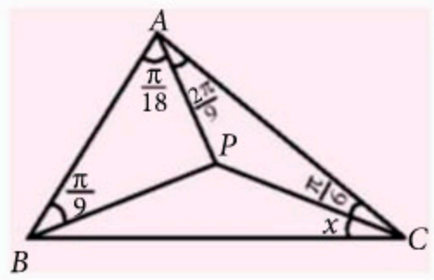
29. (b, c) : $\because \ \dfrac{r_1+r_2+r_3}{3} \ge \dfrac{3}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}$

$\therefore \quad 4R + r \ge 9r \ \Rightarrow \ 4R \ge 8r \Rightarrow R \ge 2r$

Also, $R^2 \ge 4r^2 = \dfrac{4\Delta^2}{s^2} = \dfrac{4a^2b^2c^2}{16R^2s^2} \Rightarrow R^4 \ge \dfrac{a^2b^2c^2}{4s^2}$

$$\Rightarrow \quad R^2 \geq \frac{abc}{a+b+c}$$

30. (a) : $\dfrac{PA}{\sin 20^\circ} = \dfrac{PB}{\sin 10^\circ} \Rightarrow \dfrac{PA}{PB} = \dfrac{\sin 20^\circ}{\sin 10^\circ}$



Similarly, $\dfrac{PB}{PC} = \dfrac{\sin x}{\sin(80^\circ - x)}$,

$\dfrac{PC}{PA} = \dfrac{\sin 40^\circ}{\sin 30^\circ}$

$\dfrac{PA}{PB} \times \dfrac{PB}{PC} \times \dfrac{PC}{PA} = 1$

$\Rightarrow \quad \dfrac{\sin 20^\circ}{\sin 10^\circ} \times \dfrac{\sin x}{\sin(80^\circ - x)} \times \dfrac{\sin 40^\circ}{\sin 30^\circ} = 1$

$\Rightarrow \quad \sin(80^\circ - x) = 4 \cos 10^\circ \sin 40^\circ \sin x$

$= 2\sin 50^\circ \sin x + \sin x$

$\Rightarrow \quad \sin(80^\circ - x) - \sin x = 2 \sin 50^\circ \sin x$

$\Rightarrow \quad 2 \sin(40^\circ - x) \cos 40^\circ = 2 \sin 50^\circ \sin x$

$\Rightarrow \quad \sin(40^\circ - x) = \sin x \Rightarrow x = 20^\circ$

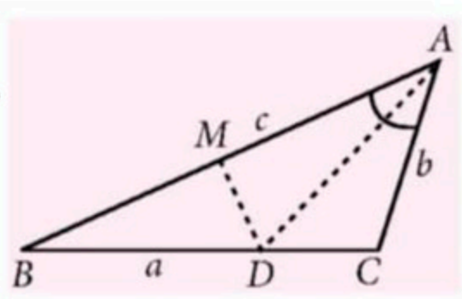
31. (b) : $\angle A = \angle C = 50^\circ$

32. (c) : $\angle ABC = 80^\circ \Rightarrow AC$ is the longest side.

33. (c) : Draw the bisector AD of $\angle BAC$.

Put $BD = x \Rightarrow AD = x$

From angle bisector theorem,



we have $\dfrac{b}{c} = \dfrac{a-x}{x}$

$\Rightarrow \quad x = \dfrac{ca}{b+c}$

Draw $DM \perp AB$. Using cos$B$, we get $a^2 - b^2 = bc$.

34. (b) : We have $a^2 = b\,(b + c) = m^2\, n^2 \Rightarrow a = mn$

Now use cos$B$.

35. (a) : Note that $C > 90^\circ$ and $A = 2B \Rightarrow 0 < B < 30^\circ$

$\Rightarrow \quad \cos 30^\circ < \cos B < 1 \Rightarrow \sqrt{3} < \dfrac{n}{m} < 2$

The least values of (n, m), satisfying this, are (7, 4).

36. A → p ; B → r ; C → q ; D → q

(A) $\sin^2 A + \sin^2 B = \sin(A + B)$

$\Rightarrow \quad 1 - \dfrac{\cos 2A + \cos 2B}{2} = \sin C$

$\Rightarrow \quad 1 - \cos(A + B) \cos(A - B) = \sin C$

$\Rightarrow \quad \cos^2 C \cos^2(A - B) + 2\cos C \cos(A - B) = \cos^2 C$

$\Rightarrow \quad \cos C = 0$

$\therefore$ Triangle is right angled.

(B) $\dfrac{bc}{2\cos A} = b^2 + c^2 - 2bc\cos A$

$\Rightarrow \quad \cos A = \dfrac{bc}{2a^2} \Rightarrow b^2c^2 = a^2(b^2 + c^2 - a^2)$

$\Rightarrow \quad (a^2 - b^2)(a^2 - c^2) = 0 \Rightarrow a = b$ or $a = c$

$\therefore$ Triangle is isosceles.

(C) $\tan\dfrac{A}{2} + \tan\dfrac{B}{2} + \tan\dfrac{C}{2} = \sqrt{3}$

$\Rightarrow \quad \left(\tan\dfrac{A}{2} - \tan\dfrac{B}{2}\right)^2 + \left(\tan\dfrac{B}{2} - \tan\dfrac{C}{2}\right)^2$

$+ \left(\tan\dfrac{C}{2} - \tan\dfrac{A}{2}\right)^2 = 0$

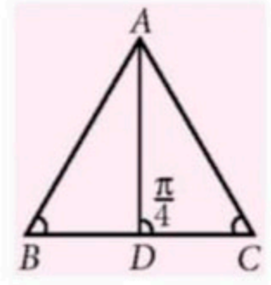
$\therefore$ Triangle is equilateral.

(D) If a, b, c are in A.P. and h_a, h_b, h_c are in A.P., where h_a, h_b, h_c are the altitudes, then $a = b = c$

$\therefore$ Triangle is equilateral.

37. (2) : From m-n cot theorem

$2\cot\dfrac{\pi}{4} = \cot B - \cot C$



$\Rightarrow \quad |\cot B - \cot C| = 2$

38. (3) : $\cos A + \cos C = 2(1 - \cos B)$

$\Rightarrow \quad 2\cos\dfrac{A+C}{2}\cos\dfrac{A-C}{2} = 2 \times 2\sin^2\dfrac{B}{2}$

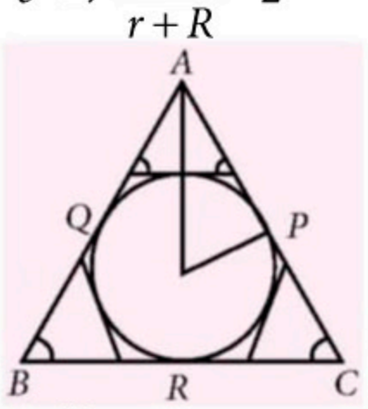
$\Rightarrow \quad \dfrac{\cos\left(\dfrac{A-C}{2}\right)}{\sin\dfrac{B}{2}} = 2 \Rightarrow \dfrac{a+c}{b} = 2 \Rightarrow a + c = 2b$

$\therefore \quad \dfrac{2s}{b} = \dfrac{3b}{b} = 3$

39. (2) : $\angle C = 90^\circ \Rightarrow 2(r + R) = a + b \Rightarrow \dfrac{a+b}{r+R} = 2$

40. (6) : $r = (s - a)\tan\dfrac{A}{2}$

$\therefore \quad \dfrac{r}{s-a} = \dfrac{r}{AP}$



$\Rightarrow \quad s - a = 5,\ s - b = 3,\ s - c = 2$

$\Rightarrow \quad s = 10$

$\Rightarrow \quad a = 5,\ b = 7,\ c = 8$

Now, $\alpha = \dfrac{ra}{s\tan\dfrac{A}{2}}$, $\beta = \dfrac{rb}{s\tan\dfrac{B}{2}}$, $\gamma = \dfrac{rc}{s\tan\dfrac{C}{2}}$

$\alpha = \dfrac{(s-a)a}{s}$, $\beta = \dfrac{(s-b)b}{s}$, $\gamma = \dfrac{(s-c)c}{s}$

$\alpha = \dfrac{5 \times 5}{10}$, $\beta = \dfrac{3 \times 7}{10}$, $\gamma = \dfrac{2 \times 8}{10}$

$\therefore \quad \alpha + \beta + \gamma = \dfrac{25 + 21 + 16}{10} = \dfrac{62}{10} \Rightarrow [\alpha + \beta + \gamma] = 6$

Monthly Test Drive - 8 CLASS XII ANSWER KEY

1.	(a)	**2.**	(a)	**3.**	(b)	**4.**	(d)
5.	(a)	**6.**	(a)	**7.**	(a, b, c)	**8.**	(a, d)
9.	(a, b)	**10.**	(b, c)	**11.**	(a, d)	**12.**	(c, d)
13.	(a)	**14.**	(b)	**15.**	(d)	**16.**	(a)
17.	(3)	**18.**	(0)	**19.**	(–4)	**20.**	(64)

CBSE warm-up!

CLASS-XI

Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Series 9 — Statistics and Probability

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

(i) All the questions are compulsory.
(ii) This question paper contains 36 questions.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION - A

Q.1 - Q.10 are multiple choice type questions. Select the correct option.

1. The mean of the first $(n + 1)$ natural numbers is

(a) $\frac{(n+2)}{2}$ (b) $\frac{(n+1)(n+2)}{2}$

(c) $\left[\frac{n(n+1)}{2}\right]^2$ (d) $\frac{(n+1)}{2}$

2. The range of the following set of observations 2, 3, 5, 9, 8, 7, 6, 5, 7, 4, 3 is

(a) 11 (b) 7 (c) 5.5 (d) 6

3. The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is

(a) 2.23 (b) 2.57 (c) 3.23 (d) 3.57

4. The mean and variance of n observations $x_1, x_2, x_3, \ldots, x_n$ are 5 and 0 respectively. If $\sum_{i=1}^{n} x_i^2 = 400$, then the value of n is equal to

(a) 80 (b) 25 (c) 20 (d) 16

5. If x and y are two variables such that $\frac{\sigma_x}{\bar{x}} > \frac{\sigma_y}{\bar{y}}$, then

(a) x is more consistent than y
(b) y is more consistent than x
(c) y is more variable than x
(d) None of these

6. A die is thrown repeatedly until a six comes up. Then, the sample space for this experiment is

(a) $S = \{6, (1, 6), (2, 6), (3, 6)\}$
(b) $S = \{(6, 1), (6, 2), (6, 4), (6, 6)\}$
(c) Infinite number of possibilities occur
(d) None of these

7. Consider the experiment of rolling a die. Let A be the event of 'getting a prime number and B be the event of 'getting an odd number', then A or B =

(a) $\{1, 2, 3, 4\}$ (b) $\{1, 2, 3, 5\}$
(c) $\{3, 5\}$ (d) $\{1, 3, 5\}$

8. Let $S = \{1, 2, 3, 4, 5, 6\}$ and $E = \{1, 3, 5\}$, then E' or $\bar{E}$ is
(a) $\{2, 4\}$ (b) $\{3, 6\}$ (c) $\{1, 2, 4\}$ (d) $\{2, 4, 6\}$

9. A single letter is selected at random from the word "FAVOURABLE". The probability that it is a vowel, is
(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{1}{2}$

10. The probability of an event A occurring is 0.5 and B occurring is 0.3. If A and B are mutually exclusive, then the probability of neither A nor B occurring is
(a) 0.9 (b) 0.7 (c) 0.1 (d) 0.2

(Q.11 - Q.15) Fill in the blanks.

11. The following information relates to a sample of size 60; $\Sigma x^2 = 18000$, $\Sigma x = 960$. The variance of the data is _____.

12. Let x_1, x_2, x_3, x_4, x_5 be the observations with mean m and standard deviation s. The standard deviation of the observations $kx_1, kx_2, kx_3, kx_4, kx_5$ is _____.

OR

For a frequency distribution variance is 4 and arithmetic mean is 16. Then the coefficient of variation is _____.

13. Event can be classified into various types on the basis of the _____ they have.

14. The probability for a randomly chosen month to have its 10^{th} day as Sunday is _____.

15. A coin is tossed twice. Then, the probability that atleast one tail occurs is _____.

OR

For every event A, there corresponds another event A' called the _____ to A.

(Q.16 - Q.20) Answer the following questions.

16. If $\frac{3}{10}$ is the probability that an event will happen, then what is the probability that it will not happen?

17. A bag contains 4 identical red balls and 3 identical black balls. The experiment consists of drawing one ball, then putting it into the bag and again drawing a ball. Then, find the possible outcomes of this experiment.

18. The runs scored by two batsmen B_1 and B_2 in their last ten matches are given :

Match	1	2	3	4	5	6	7	8	9	10
B_1	30	91	0	64	42	80	30	5	117	71
B_2	53	46	48	50	53	53	58	60	57	52

Find the range of the given distribution.

19. Given $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$. Find $P(A \text{ or } B)$, if A and B are mutually exclusive events.

20. The mean of 100 observations is 50 and their standard deviation is 5. Find the sum of squares of all the observations.

OR

There are 4 red and 3 black balls in a bag. If one ball is taken out of this bag at random, then represent the sample space and the event of this ball being black.

SECTION - B

21. What is the probability that a randomly chosen two-digit positive integer is a multiple of 3?

OR

4 cards are drawn from a well-shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?

22. The sum and the sum of squares of length x (in cm) and weight y (in g) of 30 plant products are given below:

$$\sum_{i=1}^{30} x_i = 112, \sum_{i=1}^{30} x_i^2 = 502.4, \sum_{i=1}^{30} y_i = 194 \text{ and } \sum_{i=1}^{30} y_i^2 = 1275.4$$

Which is more variable, the length or weight?

23. The probability that at least one of the events E_1 and E_2 occurs is 0.6. If the probability of the simultaneous occurrence of E_1 and E_2 is 0.2, then find $P(\bar{E}_1) + P(\bar{E}_2)$.

24. Find the variance and standard deviation of the following data :
5, 9, 10, 12, 8, 13, 6

OR

The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are 2, 4, 10, 12, 14, find the remaining observations.

25. The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given :

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40
Standard deviation	14	8	20

Which of these three subjects shows highest consistency in marks and which shows lowest?

26. In an essay competition, the odds in favour of competitors P, Q, R, S are 1 : 2, 1 : 3, 1 : 4 and 1 : 5 respectively. Find the probability that one of them wins the competition.

SECTION - C

27. Calculate the mean deviation from median of the following data :

Wages/week (in ₹)	Number of workers
10 - 20	4
20 - 30	6
30 - 40	10
40 - 50	20
50 - 60	10
60 - 70	6
70 - 80	4

28. The variance of 'n' observations is σ_1^2. If each observation is multiplied by a, then find the new variance.

OR

If $\bar{x}$ is the mean and mean deviation from mean is M.D. $(\bar{x})$, then find the number of observations lying between $\bar{x} - \text{M.D.}(\bar{x})$ and $\bar{x} + \text{M.D.}(\bar{x})$ for the following data :

34, 66, 30, 38, 44, 50, 40, 60, 42, 51

29. The following is the record of goals scored by team A in football session.

Number of goals scored by team A	0	1	2	3	4
Number of matches	1	9	7	5	3

For team B, the number of goals scored per match was 2.5, with a S.D. of 1.25. Which team may be considered more consistent?

30. A coin is tossed three times, consider the following events :

A : 'No head appears'

B : 'Exactly one head appears'

C : 'At least two heads appear'

Do they form a set of mutually exclusive and exhaustive events?

OR

A young man visits a hospital for medical checkup. The probability that he has lungs problem is 0.45, heart problem is 0.29 and either lungs or heart problem is 0.47. What is the probability that he has both types of problems: lungs as well as heart? Out of 1000 persons, how many are expected to have both types of problems?

31. In relay race there are 5 teams A, B, C, D and E.

(i) What is the probability that A, B and C finish with first, second and third respectively?

(ii) What is the probability that A, B and C are first three to finish (in any order). Assume that all finishing orders are equally likely?

32. What is the probability that in a group of 3 people at least two will have the same birthday? Assume that there are 365 days in a year and no one has his/her birthday on the 29^{th} of February.

SECTION - D

33. Three dice are thrown simultaneously. Find the probability that :

(i) All of them show the same face.

(ii) All of them show distinct faces.

(iii) Two of them show the same face.

34. Calculate mean, variance and standard deviation for the following frequency distribution.

Class	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequency	2	4	9	10	9	4	2

OR

The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases :

(i) If the wrong item is omitted.

(ii) If it is replaced by 12.

35. Let $x_1, x_2, x_3, ..., x_n$ be n values of a variable X, and let $x_i = a + hu_i$, $i = 1, 2, ..., n$, where $u_1, u_2, ..., u_n$ are the values of variable U. Then, prove that $\text{Var}(X) = h^2 \text{Var}(U)$, $h \neq 0$.

36. A box contains 100 bolts and 50 nuts. It is given that 50% bolts and 50% nuts are rusted. Two objects are selected from the box at random. Find the probability that either both are bolts or both are rusted.

OR

3 cards are drawn at random from a pack of well shuffled 52 cards. Find the probability that

(i) All the three cards are of same suit.

(ii) One is king, the other is a queen and third is a jack.

SOLUTIONS

1. **(a) :** Sum of first n natural numbers $= \dfrac{n(n+1)}{2}$

$\therefore$ Sum of first $(n+1)$ natural numbers $= \dfrac{(n+1)(n+2)}{2}$

$\therefore$ Mean $= \dfrac{(n+1)(n+2)}{2(n+1)} = \dfrac{n+2}{2}$

2. **(b):** Range = 9 – 2 = 7

3. **(b):** Mean, $\bar{x} = \dfrac{2+9+9+3+6+9+4}{7} = \dfrac{42}{7} = 6$

$$\text{M.D.} = \frac{\Sigma|x_i - \bar{x}|}{n} = \frac{4+3+3+3+0+3+2}{7} = 2.57$$

4. **(d):** Given, variance, $\sigma^2 = 0$

$$\Rightarrow \quad \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 = 0$$

$$\Rightarrow \quad \frac{1}{n}(400) - (5)^2 = 0 \Rightarrow \frac{400}{n} = 25 \Rightarrow n = 16$$

5. **(b):** Here, C.V. for y-variable is less than that for x-variable, therefore, y is more consistent than x.

6. **(c):** The sample space is
S = {6, (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (1, 1, 6), (1, 2, 6), (1, 3, 6), ...}

7. **(b):** Here, $S = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3, 5\}$ and $B = \{1, 3, 5\}$. Obviously, 'A or B' $= A \cup B = \{1, 2, 3, 5\}$

8. **(d):** Given that, $S = \{1, 2, 3, 4, 5, 6\}$ and $E = \{1, 3, 5\}$
Then, $\bar{E} = S - E = \{2, 4, 6\}$.

9. **(b):** Number of vowels are A, O, U, E
$\therefore$ One can be chosen by ${}^4C_1$ ways
Total letters in the given word = 10

$\therefore$ Required probability $= \dfrac{4}{10} = \dfrac{2}{5}$

10. **(d):** Given, $P(A) = 0.5$, $P(B) = 0.3$
$\therefore$ $P(A \cup B) = P(A) + P(B)$ $\quad (\because A \cap B = \phi)$
$= 0.5 + 0.3 = 0.8$
$\therefore$ $P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.8 = 0.2$

11. Variance $= \dfrac{\Sigma x^2}{n} - \left(\dfrac{\Sigma x}{n}\right)^2 = \dfrac{18000}{60} - \left(\dfrac{960}{60}\right)^2$.
$= 300 - 256 = 44$

12. When each item of a data is multiplied by a constant k, then S.D. is multiplied by $|k|$ and variance gets multiplied by k^2.

OR

Here, standard deviation, $\sigma = \sqrt{4} = 2$
Arithmetic mean, $\bar{x} = 16$

$\therefore$ Coefficient of variation $= \dfrac{\sigma}{\bar{x}} \times 100$

$= \dfrac{2}{16} \times 100 = \dfrac{1}{8} \times 100 = 12.5$

Hence, coefficient of variation = 12.5

13. Events can be classified into various types on the basis of the elements they have.

14. $P(10^{\text{th}}$ day as Sunday) = 1/7.

15. The sample space is $S = \{HH, HT, TH, TT\}$.
Let E be the event of getting atleast one tail

$E = \{HT, TH, TT\}$

$\therefore$ Required probability, $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{3}{4}$

OR

For every event A, there corresponds another event A' called the complementary event to A. It is also called the event 'not A''.

16. Let E be the event. Then,

$$P(E) = \frac{3}{10} \Rightarrow P(\bar{E}) = 1 - P(E) = 1 - \frac{3}{10} = \frac{7}{10}$$

Hence, $P(\text{not } E) = \dfrac{7}{10}$

17. The sample space for this experiment is
$S = \{RR, RB, BR, BB\}$, where R denotes the red ball and B denotes the black ball.

18. Range of a distribution
= Maximum value – Minimum value
$\therefore$ Range of scores of batsmen B_1 = 117 – 0 = 117
Range of scores of batsmen B_2 = 60 – 46 = 14

19. Here, $P(A) = \dfrac{3}{5}$ and $P(B) = \dfrac{1}{5}$.
Since A and B are mutually exclusive events.

$\therefore$ $P(A \cup B) = P(A) + P(B) = \dfrac{3}{5} + \dfrac{1}{5} = \dfrac{4}{5}$

20. $\Sigma x_i^{\,2} = n\{\sigma^2 + (\bar{x})^2\} = 100(5^2 + 50^2) = 252500$

OR

Let R_i denotes red balls and B_i denotes black balls.
S = Set of all possible outcomes when one ball is drawn from the bag = $\{R_1, R_2, R_3, R_4, B_1, B_2, B_3\}$
and E = Event of ball being black = $\{B_1, B_2, B_3\}$

21. Two-digit positive integers are 10, 11, 12, ... , 98, 99. These are 90 numbers out of which one number can be chosen in 90 ways.
$\therefore$ Total number of elementary events = 90.
Out of these 90 numbers, 30 numbers (12, 15, 18, ... 96, 99) are multiples of 3. One number out of these 30 numbers can be chosen in 30 ways.

$\therefore$ Favourable number of elementary events = 30

Hence, required probability $= \dfrac{30}{90} = \dfrac{1}{3}$

OR

From a pack of 52 cards, 4 cards can be drawn in ${}^{52}C_4$ ways.

There are 13 cards of diamond and 13 cards of spades.
Now, 3 cards of diamond out of 13 cards of diamond can be drawn in ${}^{13}C_3$ ways and 1 card of spade out of 13 cards of spade can be drawn in ${}^{13}C_1$ ways.
Thus, the probability of obtaining 3 diamond and 1 spade card $= \frac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4} = \frac{286}{20825}$

22. We have, $\text{Var}(x) = \frac{1}{30}\cdot\sum_{i=1}^{30} x_i^2 - \left(\frac{1}{30}\cdot\sum_{i=1}^{30} x_i\right)^2$

$= \left(\frac{502.4}{30}\right) - \left(\frac{112}{30}\right)^2 = \frac{251.2}{15} - \frac{3136}{225} = \frac{632}{225} = 2.8$

$\text{Var}(y) = \frac{1}{30}\cdot\sum_{i=1}^{30} y_i^2 - \left(\frac{1}{30}\cdot\sum_{i=1}^{30} y_i\right)^2 = \frac{1275.4}{30} - \left(\frac{194}{30}\right)^2$

$= \frac{637.7}{15} - \left(\frac{97}{15}\right)^2 = \frac{9565.5 - 9409}{225} = \frac{156.5}{225} = 0.6956$

$\therefore$ $\text{Var}(x) > \text{Var}(y)$.

$\therefore$ Length is more variable than weight.

23. Given, $P(E_1 \cup E_2) = 0.6$ and $P(E_1 \cap E_2) = 0.2$

$\therefore$ $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\Rightarrow$ $P(E_1) + P(E_2) = P(E_1 \cup E_2) + P(E_1 \cap E_2)$

$\Rightarrow$ $P(E_1) + P(E_2) = 0.6 + 0.2 = 0.8$

$\Rightarrow$ $\{1 - P(\bar{E}_1)\} + \{1 - P(\bar{E}_2)\} = 0.8$

$\Rightarrow$ $P(\bar{E}_1) + P(\bar{E}_2) = (2 - 0.8) = 1.2$

24. Here, n = number of observations = 7

and $\bar{x} = \frac{\sum x_i}{n} = \frac{5+9+10+12+8+13+6}{7} = 9$

$\sum x_i^2 = 5^2 + 9^2 + 10^2 + 12^2 + 8^2 + 13^2 + 6^2 = 619$

$\therefore$ Variance $(\sigma^2) = \frac{1}{n}\sum x_i^2 - \bar{x}^2$

$= \frac{1}{7}\cdot 619 - 9^2 = 7.43$ and $\sigma = \sqrt{7.43} = 2.72$

Thus, variance = 7.43 and standard deviation = 2.72

OR

Let a and b be the two observations.

$\therefore$ $\bar{x} = 8 \Rightarrow \frac{a+b+2+4+10+12+14}{7} = 8$

$\Rightarrow$ $a + b = 56 - 42 = 14$

Now, $\sigma^2 = 16 \Rightarrow \frac{a^2+b^2+2^2+4^2+10^2+12^2+14^2}{7} - 8^2 = 16$

$\Rightarrow$ $a^2 + b^2 = 7(64 + 16) - 460 = 100$

$\because$ $a + b = 14$, $a^2 + b^2 = 100$

$\Rightarrow$ $a = 8, b = 6$ or $a = 6, b = 8$

25. Coefficient of variation of all the three subjects are as follows:

C.V. of Mathematics $= \frac{14}{42}\times 100 = 33.33$,

C.V. of Physics $= \frac{8}{32}\times 100 = 25$

and C.V. of Chemistry $= \frac{20}{40}\times 100 = 50$

Hence, Chemistry with highest C.V. shows lowest consistency and Physics with lowest C.V. shows highest consistency.

26. Let A, B, C, D be the events that the competitors P, Q, R and S respectively win the competition. Then,

$P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P(C) = \frac{1}{5}$ and $P(D) = \frac{1}{6}$

Since, only one competitor can win the competition. Therefore, A, B , C, D are mutually exclusive events.

$\therefore$ Required probability = $P(A \cup B \cup C \cup D)$

$= P(A) + P(B) + P(C) + P(D)$ [By addition theorem]

$= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{19}{20}$

27. *Refer to answer 17, page no. 324 of MTG CBSE Champion Mathematics, Class-11*

28. Let the 'n' observations be x_1, x_2, ..., x_n and $\bar{x}_1$ be their mean.

Then, variance, $\sigma_1^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x}_1)^2$ ...(i)

If each observation is multiplied by a, then the new mean $\bar{x}_2$ is given by $\bar{x}_2 = \frac{1}{n}\Sigma(x_i\, a) = \frac{a}{n}\Sigma x_i = a\bar{x}_1$ ...(ii)

$\therefore$ The new variance, $\sigma_2^2(\text{say}) = \frac{1}{n}\Sigma(a\, x_i - \bar{x}_2)^2$

$$= \frac{1}{n}\Sigma a^2(x_i - \bar{x}_1)^2 \qquad \text{[From (ii)]}$$

$$= a^2 \frac{1}{n}\Sigma (x_i - \bar{x}_1)^2 = a^2\sigma_1^2 \qquad \text{[From (i)]}$$

Thus, the new variance is $a^2\sigma_1^2$.

OR

Refer to answer 23, page no. 325 of MTG CBSE Champion Mathematics, Class-11

29. Let the assumed mean for team $A = 2$.

No. of goals by team A (x_i)	No. of matches (f_i)	$d_i = x_i - a = x_i - 2$	$f_i d_i$	$f_i d_i^2$
0	1	–2	–2	4
1	9	–1	–9	9
2	7	0	0	0
3	5	1	5	5
4	3	2	6	12
	$\Sigma f_i = 25$		$\Sigma f_i d_i = 0$	$\Sigma f_i d_i^2 = 30$

$$\text{A.M.} = \bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 2 + \frac{0}{25} = 2$$

$$\text{S.D.} = \sigma = \sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2} = \sqrt{\frac{30}{25}} = \sqrt{\frac{6}{5}} = 1.1$$

For team *A* : Coefficient of variation $= \frac{1.1}{2} \times 100 = 55$

For team *B* : Coefficient of variation $= \frac{1.25}{2.5} \times 100 = 50$

Clearly 55 > 50. Hence team B is more consistent.

30. If S be the sample space of tossing a coin three times then $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Now, A (Event when no head appears) $= \{TTT\}$

B (Event when exactly one head appears) $= \{HTT, THT, TTH\}$

C (Event when atleast two heads appear) $= \{HHT, HHH, HTH, THH\}$

Clearly, $A \cap B \cap C = \phi$

So, A, B and C are mutually exclusive events.

Clearly, $A \cup B \cup C = S$

So, A, B and C are exhaustive events.

OR

Let L represents the lungs problem and H represents the heart problem.

Given, $P(L) = 0.45$, $P(H) = 0.29$ and $P(L \cup H) = 0.47$

$\because \quad P(L \cup H) = P(L) + P(H) - P(L \cap H)$

$\Rightarrow \quad 0.47 = 0.45 + 0.29 - P(L \cap H)$

$\Rightarrow \quad P(L \cap H) = 0.45 + 0.29 - 0.47 = 0.27$

$\therefore$ The probability that he has both types of problems is 0.27. Out of 1000 persons, number of people having both types of problems $= 0.27 \times 1000 = 270$

31. *Refer to answer 87, page no. 365 of MTG CBSE Champion Mathematics, Class-11*

32. Let S be the sample space and E be the event that "at least two people have the same birthday"

$\therefore \quad E'$ = all the three people have distinct birthdays

Then, $n(S)$ = number of ways in which three persons may have their birthdays $= 365 \times 365 \times 365 = 365^3$

and $n(E') = 365 \times 364 \times 363$

$$\text{Now, } P(E') = \frac{n(E')}{n(S)} = \frac{365 \times 364 \times 363}{365^3} = \frac{364 \times 363}{365^2}$$

Hence, required probability,

$$P(E) = 1 - P(E') = 1 - \frac{364 \times 363}{365^2}$$

33. The total number of elementary events associated to the random experiment of throwing three dice simultaneously is $6 \times 6 \times 6 = 6^3$.

(i) All dice show the same face in one of the following mutually exclusive ways :

(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)

So, favourable number of elementary events = 6.

$$\text{Hence, required probability} = \frac{6}{6^3} = \frac{1}{36}$$

(ii) The total number of ways in which all dice show different faces is equal to the number of ways of arranging 6 distinct objects by taking three at a time *i.e.*, ${}^6C_3 \times 3!$.

So, favourable number of elementary events $= {}^6C_3 \times 3!$

$$\text{Hence, required probability} = \frac{{}^6C_3 \times 3!}{6^3} = \frac{5}{9}$$

(iii) Select a number which occurs on two dice out of the six numbers 1, 2, 3, 4, 5, 6 marked on the six faces of a die. This can be done in ${}^6C_1$ ways. Now, we have three numbers like 1, 1, 2 ; 2, 2, 5 etc. These three digits can be arranged in $\frac{3!}{2!}$ ways.

So, the favourable number of elementary events $= {}^6C_1 \times {}^5C_1 \times \frac{3!}{2!}$

$$\text{Hence, required probability} = \frac{90}{6^3} = \frac{5}{12}$$

34. Here, $h = 30$. Let the assumed mean be $a = 105$.

Now, we prepare the table given below.

Class	Frequency (f_i)	Mid-point (x_i)	$y_i = \frac{(x_i - 105)}{30}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0-30	2	15	–3	9	–6	18
30-60	4	45	–2	4	–8	16
60-90	9	75	–1	1	–9	9
90-120	10	105 = a	0	0	0	0
120-150	9	135	1	1	9	9
150-180	4	165	2	4	8	16
180-210	2	195	3	9	6	18
Total	$N = 40$				0	86

$\therefore$ $a = 105, h = 30, N = \sum f_i = 40, \sum f_i y_i = 0$ and $\sum f_i y_i^2 = 86$

$\therefore$ Mean, $\bar{x} = \left(a + \frac{\sum f_i y_i}{N} \times h\right)$

$= \left(105 + \frac{0}{40} \times 30\right) = 105$

Variance, $\sigma^2 = \frac{h^2}{N^2} \cdot \{N \cdot \sum f_i y_i^2 - (\sum f_i y_i)^2\}$

$= \frac{(30)^2}{(40)^2} \cdot \{40 \times 86 - (0)^2\} = 1935$

Standard deviation, $\sigma = \sqrt{1935} = 43.98$.

OR

Refer to answer 60, page no. 334 of MTG CBSE Champion Mathematics, Class-11

35. We have, $x_i = a + h\,u_i$, $i = 1, 2, ..., n$

$\Rightarrow \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} (a + h\,u_i) = na + h \sum_{i=1}^{n} u_i$

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} x_i = a + h\left(\frac{1}{n}\sum_{i=1}^{n} u_i\right)$

$\Rightarrow \bar{X} = a + h\,\bar{U} \quad \left[\because \bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i \text{ and } \bar{U} = \frac{1}{n}\sum_{i=1}^{n} u_i\right]$

$\therefore$ $x_i - \bar{X} = (a + h\,u_i) - (a + h\,\bar{U})$, $i = 1, 2, ..., n$

$\Rightarrow x_i - \bar{X} = h\,(u_i - \bar{U})$, $i = 1, 2, ..., n$

$\Rightarrow (x_i - \bar{X})^2 = h^2 (u_i - \bar{U})^2$, $i = 1, 2, ..., n$

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{X})^2 = h^2 \left\{\frac{1}{n}\sum_{i=1}^{n} (u_i - \bar{U})^2\right\}$

$\Rightarrow$ Var $(X) = h^2$ Var (U).

36. Total number of objects = (100 + 50) = 150

Let S be the sample space. Then,

$n(S)$ = Number of ways of selecting 2 objects out of 150

$= {}^{150}C_2$

Number of rusted objects

= (50% of 100) + (50% of 50) = (50 + 25) = 75

Let E_1 = Event of selecting 2 bolts out of 100 bolts

and E_2 = Event of selecting 2 rusted objects out of 75 rusted objects.

$\therefore$ $(E_1 \cap E_2)$ = Event of selecting 2 rusted bolts out of 50 rusted bolts

$\therefore$ $n(E_1)$ = Number of ways of selecting 2 bolts out of $100 = {}^{100}C_2$

$\therefore$ $n(E_2)$ = Number of ways of selecting 2 rusted objects out of $75 = {}^{75}C_2$

$\therefore$ $n(E_1 \cap E_2)$ = Number of ways of selecting 2 rusted bolts out of $50 = {}^{50}C_2$

$\therefore$ $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{{}^{100}C_2}{{}^{150}C_2}$, $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{{}^{75}C_2}{{}^{150}C_2}$

and $P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{{}^{50}C_2}{{}^{150}C_2}$

P(selecting both bolts or both rusted objects)

$= P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{{}^{100}C_2}{{}^{150}C_2} + \frac{{}^{75}C_2}{{}^{150}C_2} - \frac{{}^{50}C_2}{{}^{150}C_2} = \frac{({}^{100}C_2 + {}^{75}C_2 - {}^{50}C_2)}{{}^{150}C_2}$

$= \frac{(4950 + 2775 - 1225)}{11175} = \frac{6500}{11175} = \frac{260}{447}$

Hence, the required probability is $\frac{260}{447}$.

OR

Refer to answer 95, page no. 366 of MTG CBSE Champion Mathematics, Class-11

SOLUTION SENDERS

Mathdoku (November)

- Pravin Bajirao Thite (Maharashtra)
- Mukul Prajapat (Rajasthan)
- Neha Sonak (Haryana)
- Vaibhav Bhartiya (Bihar)
- Samir Das

Mathdoku (December)

- Mukul Prajapat (Rajasthan)
- Soumyajoy Kundu (West Bengal)

Maths Musing (Set - 203)

- Gouri Sankar Adhikary (West Bengal)

Samurai Sudoku (November)

- Shailaja Tekaday (Maharashtra)
- Vaibhav Bhartiya (Bihar)
- B. Aakash (Tamil Nadu)
- Sadhvi Vinayak Deshpande (Karnataka)
- Varun Chaturvedi (Gujarat)

Samurai Sudoku (December)

- Nikhil Raj (Jharkhand)
- Subhranil Pan (West Bengal)
- Ashish Singh Chauhan (Dadra and Nagar Haveli)
- Aviral Vishwakarma (Delhi)
- G. Dilipraj (Telangana)

MONTHLY TEST DRIVE

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 8 : Straight Lines

Time Taken : 60 Min.

Only One Option Correct Type

1. A straight line through the origin meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at points P and Q respectively. Then, the point O divides the segment PQ in the ratio

(a) 1 : 2 (b) 3 : 4 (c) 2 : 1 (d) 4 : 3

2. The combined equation of the lines L_1 and L_2 is $2x^2 + 6xy + y^2 = 0$ and that of the lines L_3 and L_4 is $4x^2 + 18xy + y^2 = 0$. If the angle between L_1 and L_4 be α, then the angle between L_2 and L_3 will be

(a) $\frac{\pi}{2} - \alpha$ (b) 2α (c) $\frac{\pi}{4} + \alpha$ (d) α

3. If the equation $8x^2 + 8xy + 2y^2 + 26x + 13y + 15 = 0$ represents a pair of straight lines, then the distance between them is

(a) $\frac{7}{\sqrt{5}}$ (b) $\frac{7}{2\sqrt{5}}$ (c) $\sqrt{\frac{7}{5}}$ (d) $\frac{\sqrt{7}}{5}$

4. A square of side 'a' lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha(0 < \alpha < \pi/4)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is

(a) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$
(b) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$
(c) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$
(d) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$

5. All chords of the curve $3x^2 - y^2 - 2x + 4y = 0$, which subtend a right angle at the origin always pass through the point

(a) (1, 2) (b) (−1, 2) (c) (1, −2) (d) (−1, −2)

6. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is

(a) $y = x + \sqrt{3}$ (b) $\sqrt{3}y = x - \sqrt{3}$
(c) $y = \sqrt{3}x - \sqrt{3}$ (d) $\sqrt{3}y = x - 1$

One or More Than One Option(s) Correct Type

7. Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point (1, −10). Then the possible equation of third side is

(a) $x - 3y - 31 = 0$ (b) $3x + y + 7 = 0$
(c) $x + 2y + 19 = 0$ (d) $2x + y + 8 = 0$

8. If $a^2 + 9b^2 = 6ab + 4c^2$, then the straight line $ax + by + c = 0$ passes through

(a) $\left(\frac{-1}{2}, \frac{3}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{-3}{2}\right)$

PUZZLE CORNER

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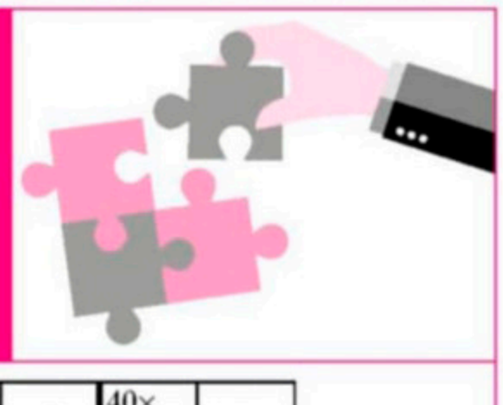

18× 6	3	7+ 5	2	40× 1	4
1	2− 4	6	18× 3	2	5
60× 5	2× 2	1	6	12× 4	3
3	1	9+ 4	5	8+ 6	2
4	60× 5	6× 2	3+ 1	3	6+ 6
2	6	3	1− 4	5	1

(c) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (d) $\left(\frac{-1}{2}, \frac{-3}{2}\right)$

9. A line which makes an acute angle θ with the positive direction of the x-axis is drawn through the point $P(3, 4)$ to meet the line $x = 6$ at R and $y = 8$ at S. Then,

(a) $PR = 3\sec\theta$ (b) $PS = 4\,\text{cosec}\,\theta$

(c) $PR + PS = \dfrac{2(3\sin\theta + 4\cos\theta)}{\sin 2\theta}$

(d) $\dfrac{9}{(PR)^2} + \dfrac{16}{(PS)^2} = 1$

10. Let $0 < p < q$ and $a \neq 0$ such that the equation $px^2 + 4\lambda xy + qy^2 + 4a(x + y + 1) = 0$ represents a pair of straight lines, then a can lie in the interval

(a) $(-\infty, \infty)$ (b) $(-\infty, p]$ (c) $[p, q]$ (d) $[q, \infty)$

11. The lines joining the origin to the point of intersection of $3x^2 + mxy - 4x + 1 = 0$ and $2x + y - 1 = 0$ are at right angles. Then, which of the following is not a possible value of m?

(a) -4 (b) 4 (c) 7 (d) 3

12. Two lines from the family of lines $(1 + 2\lambda)x - (1 + \lambda)y + 1 = 0$ and the line $x + y = 5$ form an equilateral triangle. Then, the equation of the two lines can be

(a) $y-2=(2+\sqrt{3})(x-1)$ (b) $y-2=(2-\sqrt{3})(x-1)$

(c) $y-2=(3-\sqrt{2})(x-1)$ (d) $y-1=(3-\sqrt{2})(x-1)$

13. Two pairs of straight lines have the equations $y^2 + xy - 12x^2 = 0$ and $ax^2 + 2hxy + by^2 = 0$. One line will be common among them if

(a) $a + 8h - 16b = 0$ (b) $a - 8h + 16b = 0$

(c) $a - 6h + 9b = 0$ (d) $a + 6h + 9b = 0$

Comprehension Type

The equation of an altitude of an equilateral triangle is $\sqrt{3}x + y = 2\sqrt{3}$ and one of the vertices is $(3, \sqrt{3})$.

14. The possible number of triangles is

(a) 1 (b) 2 (c) 3 (d) 4

15. Which of the following is not one of the possible vertices of the triangle?

(a) $(0, 0)$ (b) $(0, 2\sqrt{3})$

(c) $(3, -\sqrt{3})$ (d) None of these

Matrix Match Type

16. Consider the lines given by

$L_1 : x + 3y - 5 = 0$

$L_2 : 3x - ky - 1 = 0$

$L_3 : 5x + 2y - 12 = 0$

Match the following columns:

Column-I		Column-II	
P.	L_1, L_2, L_3 are concurrent if	1.	$k = -9$
Q.	One of L_1, L_2, L_3 is parallel to at least one of the other two if	2.	$k = -\frac{6}{5}$
R.	L_1, L_2, L_3 form a triangle if	3.	$k = \frac{5}{6}$
S.	L_1, L_2, L_3 do not form a triangle if	4.	$k = 5$

	P	Q	R	S
(a)	2, 3	4	1	1, 3
(b)	1, 2	3	4	2
(c)	4	1, 2	3	1, 2, 4
(d)	2, 4	1, 3	2	3

Numerical Value Type

17. In ΔABC, the coordinates of A are $(1, 2)$, $y = x$ is the perpendicular bisector of the side AB and $x - 2y + 1 = 0$ is the equation of the internal angle bisector of $\angle C$. If the equation of the side BC is $ax + by - 5 = 0$, then the value of $a - b$ is

18. If the line joining the origin to point of intersection of curve (C) and line (L) are at right angle, where

$C : 3x^2 - xy + 3y^2 + 2x - 3y + 1 = 0$

$L : 2x + 3y = k$, then the value of $5k - 6k^2$ is

19. If one of the lines represented by the equation $mx^2 - (1 - m^2)\,xy - my^2 = 0$ bisects two angles between the coordinate axes, then $|m|$ is equal to

20. The area (in sq. units) of the triangle formed by the line $x + y = 3$and the angle bisectors of the pair of lines $x^2 - y^2 + 2y - 1 = 0$ is

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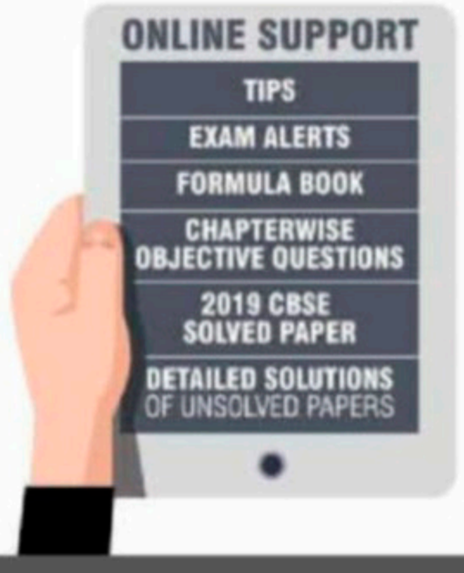

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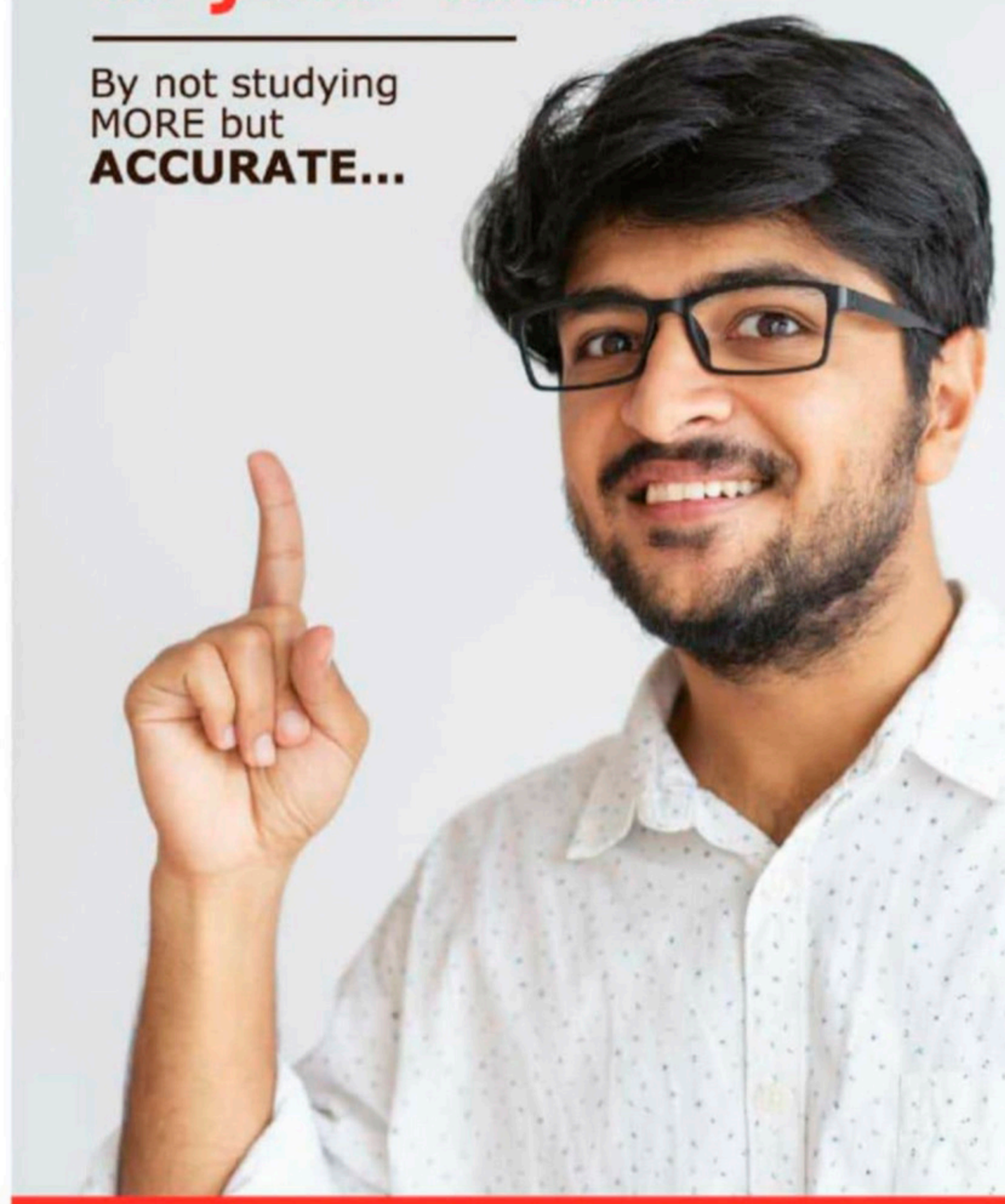

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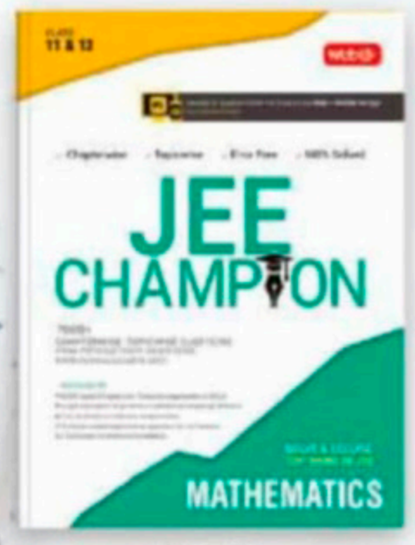

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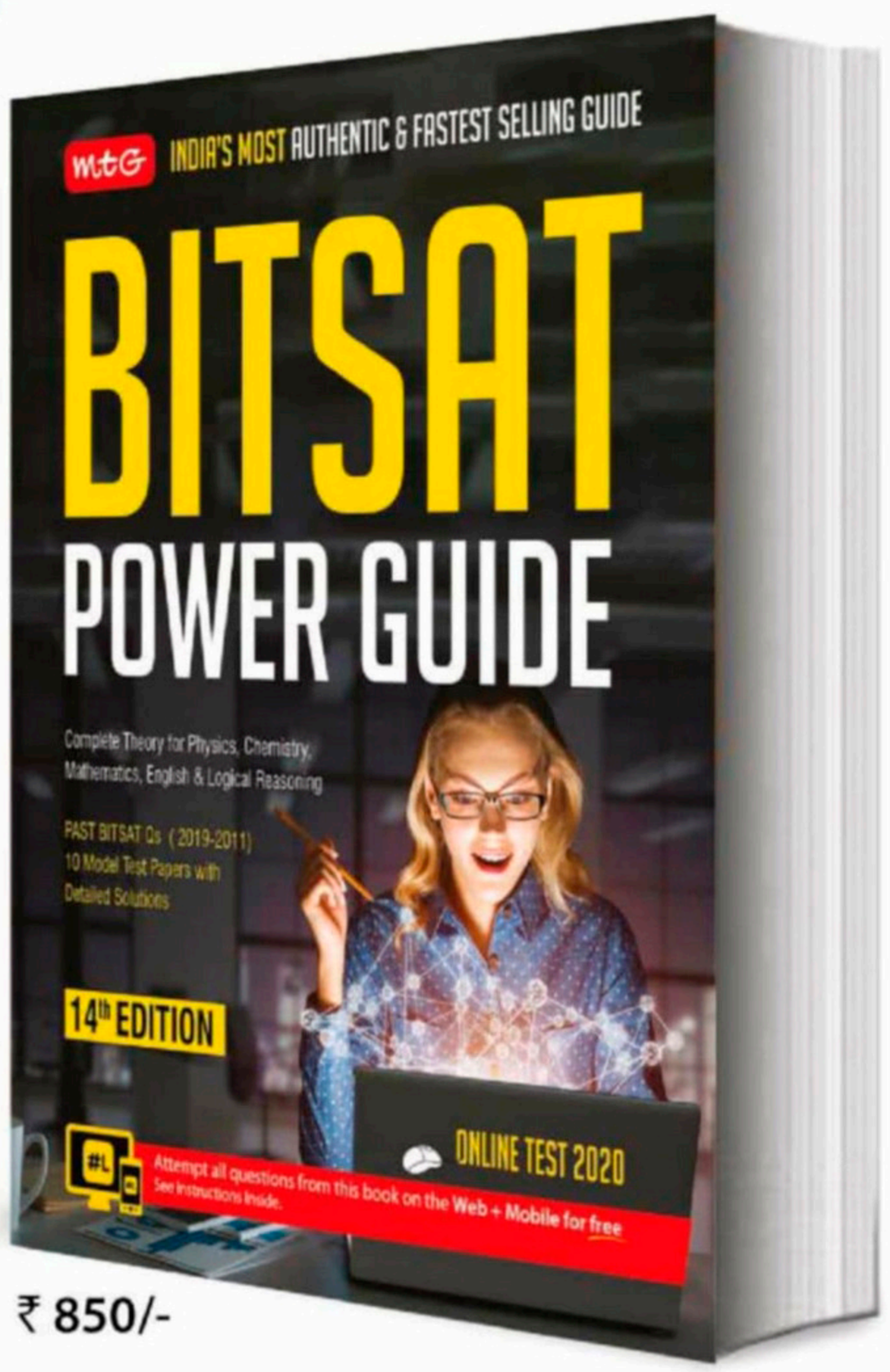

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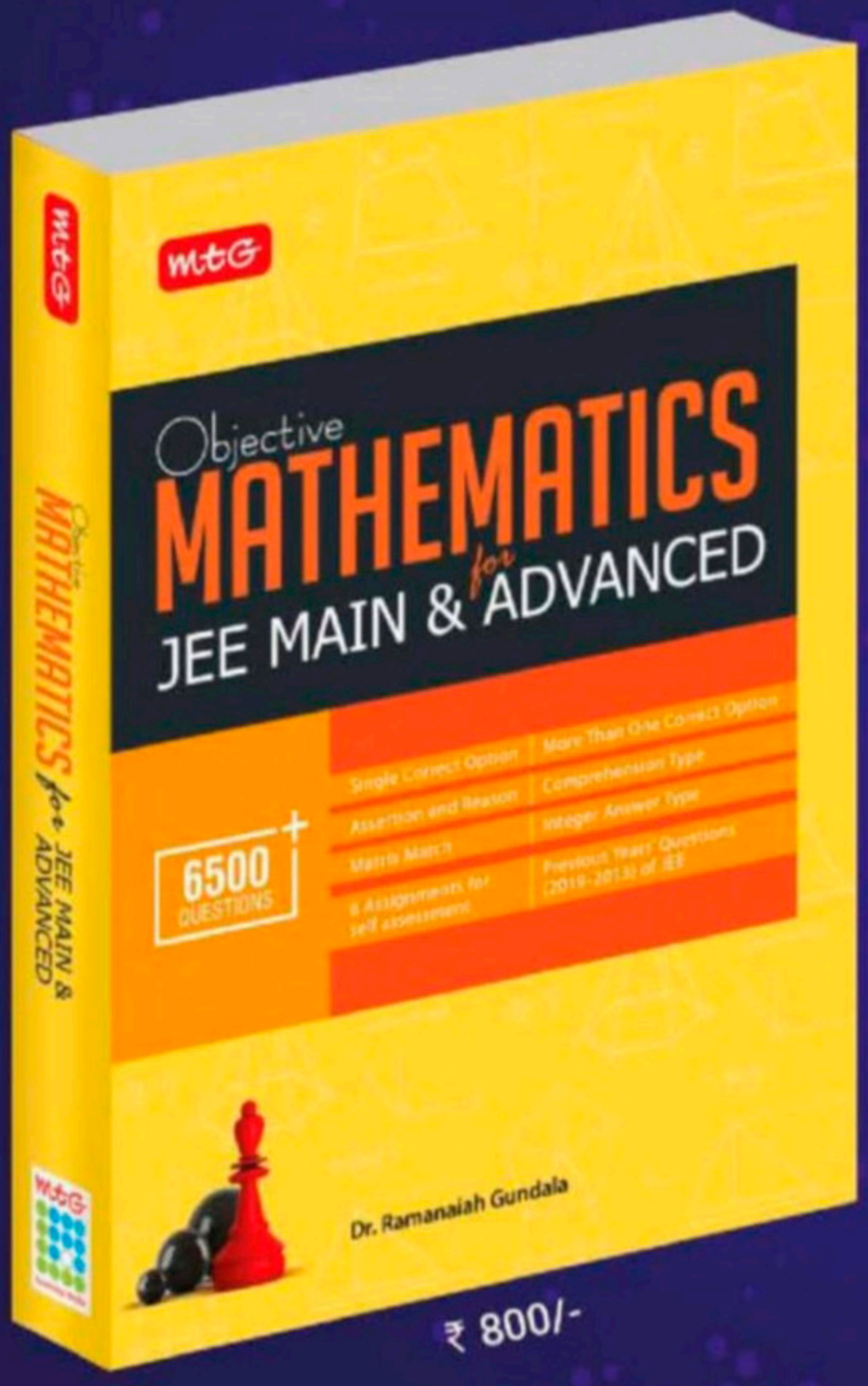

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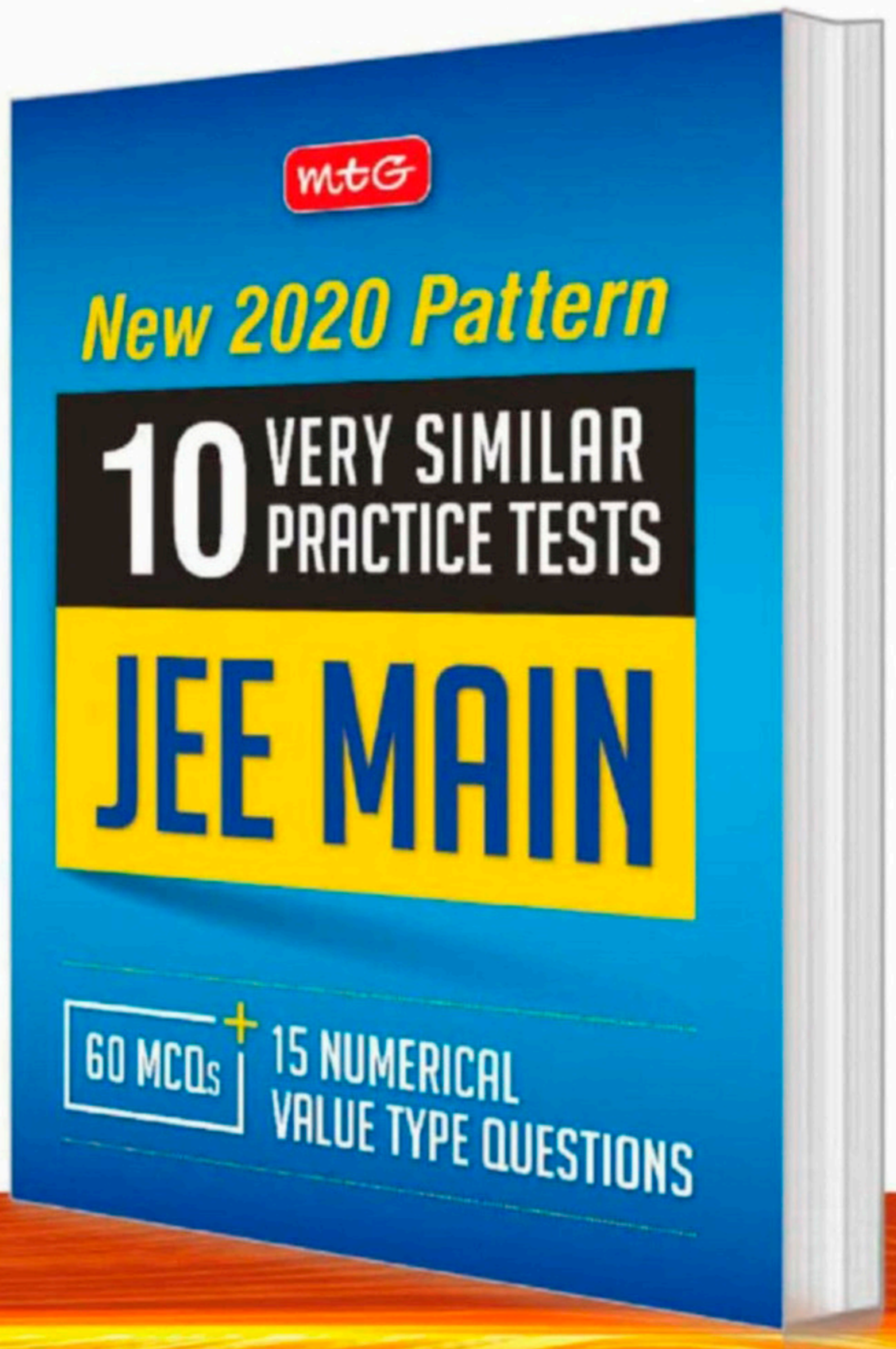

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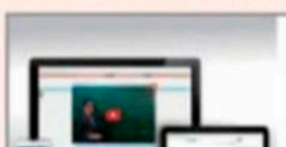
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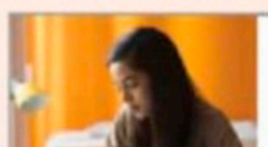
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